

Empirical and Theoretical Analysis of Public Procurement  
Auctions

DISSERTATION

Presented in Partial Fulfillment of the Requirements for  
the Degree Doctor of Philosophy in the  
Graduate School of The Ohio State University

By

Jun Nakabayashi, M.A.

\* \* \* \* \*

The Ohio State University

2009

Dissertation Committee:

Howard P. Marvel, Adviser

Lixin Ye

Matthew S. Lewis

Approved by

---

Adviser

Graduate Program in  
Economics

UMI Number: 3375830

### INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

UMI<sup>®</sup>

---

UMI Microform 3375830  
Copyright 2009 by ProQuest LLC  
All rights reserved. This microform edition is protected against  
unauthorized copying under Title 17, United States Code.

---

ProQuest LLC  
789 East Eisenhower Parkway  
P.O. Box 1346  
Ann Arbor, MI 48106-1346

Dedicated to my wife Shinobu

## ABSTRACT

Spending approximately 10 percent of the gross domestic product (GDP), governments are the biggest buyer of goods and services for many countries. Sound public procurement policies and practices are essential not only to good governance, but also to a strong and stable economy. On the other hand, due to the fact that more than 99 percent of all businesses are small in most of the countries, the involvement of small businesses are vital for the governments seeking to establish competition, innovation and value for money in the delivery of public goods and services. In fact, small businesses are likely to be disadvantage in production costs and lack in knowledge of available contract opportunities. Moreover, since it naturally would fit better to the small businesses, many of them serve to government contracts as subcontractor. In this thesis, I analyze public procurement auctions both empirically and theoretically, focusing on the role and impact of small businesses in government procurement market.

To level the playing field for businesses seeking to bid for public sector contracts, many governments adopt small business programs and provide contract opportunities for businesses operated by members of disadvantaged groups. The federal government, for instance, has its goal of awarding 23 percent of its contracting dollars to small businesses. The redistribution arising from such programs, however, can introduce significant added costs to government procurement budgets. In

my first essay “Small Business Set-asides in Procurement Auctions: An Empirical Analysis,” I examine to what extent small business set-asides increase government procurement costs, and how set-asides promote access of the beneficiaries to procurement markets. The estimates employ data on Japanese public construction projects, where approximately 50 percent of the procurement budget is set-aside for small and medium enterprises (SMEs). Using nonparametric estimation of asymmetric first-price auctions with affiliated private values, I find that, in each auction, smaller firms are likely to have higher production costs and obtain less expected profits than larger firms. Applying such a quantitative relationship between profitability and firm size into the static entry model, I then conduct a counterfactual simulation to indicate that approximately 40 percent of SMEs would exit from the procurement market were set-asides to be removed. Surprisingly, the resulting lack of competition would increase government procurement costs (0.4 percent), more than offsetting the resulting production cost inefficiency.

The second essay “Procurement auctions with pre-award subcontracting” addresses issues in pre-award subcontract competition. To be the lowest bidder in procurement auctions, prime contractors commonly solicit bids from subcontractors at the bid preparation stage. A remarkable feature of the subcontract competition is that winning is not everything; the awarded subcontractor gets a job conditional on his prime contractor’s successful bid. I model a simple two-stage game. Each prime contractor solicits irrevocable price quotes from subcontractors and chooses one in the first stage, assuming that the subcontractors’ costs are private information. Relying on the selected price quote, the prime contractor computes the project cost and bids in the auction at the second stage. I find that, in response to increasing competition in the downstream competition, prime contractors have

a stronger bargaining power against subcontractors. The behavior results in an endogenous downward shift in the distribution of bidders' costs as the number of rivals increases, or the reservation price drops, unlike the case in the standard mechanism design model where the distribution of bidder's private information is independent from such competitive environment variables. As a result of the theory, I demonstrate that the revenue maximizing reservation price is decreasing in the number of bidders. Furthermore, if the prime contractors' endogenous participation in the procurement auction is taken into account, it is shown that subsidizing the potential bidders' entry is a remedy to solve the double marginalization problem, allowing the auctioneer to extract more rents from subcontractors.

The final essay "Equilibria in Asymmetric Auctions with Entry" discusses an affiliated private value auction with entry. The contribution of this paper is to relax the symmetric assumption (i.e., the potential bidders may not be ex ante the same). The main findings are threefold. First, auction is optimal (revenue maximizing) if and only if the mechanism is ex post efficient. Second, without any participation control, a coordination problem in which only the lower value bidders participate and the higher value bidders stay out is likely, which makes the auctioneer worse off. Finally, there is an entry fee/subsidization scheme which, together with an ex post efficient mechanism, implements the optimal outcome as a unique equilibrium. Contrary to the existing theorem which claims that in asymmetric auctions well-designed ex post inefficient mechanisms are optimal (e.g., Myerson (1981), McAfee and McMillan (1987)), our results show that, even in an asymmetric auction, the mechanisms with free entry and no distortion are optimal taking into account the potential bidders endogenous participation.

## ACKNOWLEDGMENTS

I owe my deepest gratitude to my advisor, Howard P. Marvel for his guidance, understanding, and patience during my graduate studies. I am especially grateful for his assistance in helping me to organize my ideas into workable research questions. He also taught me how to question and express ideas which I believe the inevitable things as a good researcher. I hope that one day, I would become as good an advisor to my students as Howard has been to me.

I wish to thank my co-advisor, Lixin Ye for his assistance and guidance to getting my graduate career started on the right foot and providing me with the foundation for studying auctions and mechanism design. I am especially grateful for his help in finding references from both theoretical and empirical auction and mechanism design literature. I would also like to thank Matthew Lewis for his suggestions regarding estimation and interpretation of econometric results.

Additionally, I wish to thank David Blau, Sukehiro Hosono for their insightful comments and suggestions in and in particular, Lung-fei Lee for very helpful suggestions and comments on the empirical methodologies in my dissertation.

Finally, and most importantly, I would like to thank my wife Shinobu for her encouragement, infinite patience and unconditional love. Her support and understanding helped me overcome many difficult situations and finish this dissertation.

## VITA

June 27, 1973 ..... Born - Tokyo, Japan

1997 ..... B.A. Business, Hitotsubashi University

2003 ..... M.A. Economics,  
Kwansei Gakuin University

2004 ..... M.A. Economics,  
The Ohio State University

2005-present ..... Graduate Teaching Associate,  
The Ohio State University.

## FIELDS OF STUDY

Major Field: Economics



## TABLE OF CONTENTS

	Page
Abstract . . . . .	iii
Acknowledgments . . . . .	vi
Vita . . . . .	vii
List of Tables . . . . .	xi
List of Figures . . . . .	xii
Chapters:	
1. Introduction . . . . .	1
1.1 Introduction . . . . .	1
1.2 Literature review . . . . .	5
1.2.1 Small business programs . . . . .	5
1.2.2 Subcontracting . . . . .	7
1.2.3 Auction with Entry . . . . .	7
2. Small Business Set-asides in Procurement Auctions: An Empirical Analysis . . . . .	9
2.1 Introduction . . . . .	10
2.2 Public construction markets in Japan . . . . .	15
2.2.1 Overview . . . . .	15
2.2.2 Preliminary qualification examination . . . . .	17
2.2.3 Set-asides in the public construction market . . . . .	19
2.3 The data . . . . .	20
2.3.1 Overview . . . . .	20
2.3.2 Summary statistics of bids and scores . . . . .	24

2.4	Recovery of the bidders' cost distribution . . . . .	31
2.4.1	Overview . . . . .	31
2.4.2	A model of asymmetric auctions . . . . .	32
2.4.3	Nonparametric estimation . . . . .	35
2.5	A model for auctions with entry . . . . .	39
2.5.1	Analysis for the auction stage . . . . .	42
2.5.2	Analysis for an entry equilibrium . . . . .	44
2.5.3	An empirical model for auctions with entry . . . . .	45
2.6	Simulation . . . . .	51
2.6.1	The model . . . . .	51
2.6.2	Results . . . . .	54
2.7	Conclusion . . . . .	57
3.	Procurement Auctions with Pre-award Subcontracting . . . . .	60
3.1	Introduction . . . . .	61
3.2	The model . . . . .	66
3.3	Equilibrium in upstream auctions . . . . .	68
3.3.1	A simple case . . . . .	69
3.3.2	Generalized cases . . . . .	71
3.4	The downstream auction . . . . .	80
3.4.1	The theory of optimal reservation price . . . . .	88
3.4.2	The theory of auctions with endogenous participation . . . . .	88
3.5	Discussion . . . . .	91
3.6	Conclusion . . . . .	93
4.	Equilibria in Asymmetric Auctions with Entry . . . . .	95
4.1	Introduction . . . . .	96
4.2	Model . . . . .	100
4.3	The Nash bidding strategy in the asymmetric auctions . . . . .	102
4.4	Type-symmetric entry equilibria . . . . .	103
4.5	The property of the entry equilibria . . . . .	105
4.5.1	Affiliated private value . . . . .	116
4.6	Discussion . . . . .	117
4.7	Conclusion . . . . .	119
5.	Conclusion . . . . .	121
	Appendices . . . . .	124

Appendix for chapter 2 . . . . .	125
2.A An alternative proof for Zhang and Guler (2005) . . . . .	125
2.B Linearization of $\Lambda(\cdot)$ . . . . .	127
2.C Linearization of $\Gamma(\cdot)$ . . . . .	128
Appendix for chapter 3 . . . . .	130
3.A An alternative way to obtain $1 - F_c(\cdot)$ . . . . .	130
3.B Proof that $F_c$ has IHR . . . . .	131
3.C Proof that $Q$ has IHR . . . . .	131
Appendix for chapter 4 . . . . .	134
4.A Proof for Lemma 4 . . . . .	134
4.B Proof for Lemma 5 . . . . .	135
4.C Proof for Theorem 2 . . . . .	135
4.D Proof for Proposition 9 . . . . .	137
4.E Proof for Proposition 10 . . . . .	138
References . . . . .	138

## LIST OF TABLES

Table	Page
2.1 Projects Yearly (¥million) . . . . .	22
2.2 Projects from FY2005 through FY2007 (¥million) . . . . .	22
2.3 Summary statistics : The business evaluation score of actual bidders .	25
2.4 Summary Statistic on Project Size . . . . .	27
2.5 Regression results of normalized bids and estimated costs . . . . .	31
2.6 Regression Result for bid margins . . . . .	40
2.7 Project category . . . . .	46
2.8 Regression Result for linear probability model . . . . .	48
2.9 Regression Result for expected payoffs . . . . .	50
2.10 Regression Result for lowest bids . . . . .	53
2.11 Estimation for the effect of set-asides . . . . .	56

## LIST OF FIGURES

Figure	Page
2.1 Data area . . . . .	23
2.2 Normalized score of actual bidders . . . . .	26
2.3 Project Size ( $\log_{10}$ of engineer's estimate) . . . . .	27
2.4 Densities (Percentage bids) . . . . .	29
2.5 BE score of actual bidders . . . . .	30
2.6 BE score of all the firms on the certified contractor lists . . . . .	30
2.7 Few bidders: $n = 5$ . . . . .	39
2.8 Many bidders: $22 \leq n \leq 28$ . . . . .	39
2.9 The model of auctions with entry . . . . .	41

# CHAPTER 1

## INTRODUCTION

### 1.1 Introduction

Spending 9-13 percent of the gross domestic product (GDP), governments are the biggest buyer of goods and services for many countries. Sound public procurement policies and practices are essential to good governance and a strong and stable economy. On the other hand, due to the fact that more than 99 percent of all businesses are small in most of the countries, the involvement of small businesses are vital for governments who are seeking to establish competition, innovation and value for money in their procurement. In this dissertation, I analyze government procurement auctions both empirically and theoretically, focusing on the role and impact of small businesses 1) as prime contractors 2) as subcontractors in government procurement markets.

As prime contractors, or bidders in the government procurement auctions, small businesses are likely to be disadvantage in production costs and to be lack in knowledge of available contract opportunities. To promote their access to and secure a fair share in procurement markets, setting aside (or reserving) contracts is a widespreadly

used way. The federal government, for instance, has its goal of awarding 23 percent of its contracting dollars to small businesses. The redistribution arising from such programs, however, can introduce significant added costs to government procurement budgets. The essay “Small Business Set-asides in Procurement Auctions: An Empirical Analysis” empirically measures the impact of the affirmative action program on government procurement costs.

One of the methodological innovations of this research lies in the use of empirical model for asymmetric auctions. The relevant nonparametric estimation technique has been developed in the last five years, and only a few applied papers exist in the literature. In addition, there still remain many open questions in the theoretical model of asymmetric auctions. Nonetheless, the analysis for asymmetric auctions in this research reached remarkable results. Assuming that firm size affects the bidder’s advantage in production costs, the empirical analysis showed that it significantly affects their bidding behaviors as predicted in the theoretical literature. Furthermore, the research employed a unique approach to avoid the kernel dimensionality going to infinity, supposing that each bidder partially knows the opponent bidders. Taking into account that bidders’ names keep secret until the auction is over in reality, this assumption not only helps the empirical procedure be implementable, but also makes the model assumption more realistic. Another innovation is found in that the counterfactual simulation of this research is based on a model of auctions with entry. Contrary to standard auction models where the number of bidders is given, the endogenous entry model considers that it varies according to the bidder’s participation decision. It is theoretically well-known that several key features of the auction model with the fixed number of bidders are altered if entry

is taken into account, e.g. the bidder's expected profit is driven down to zero. On the basis of this setting, the simulation study brought a counterintuitive result that set-asides not only improve equity between large and small firms but also help save public spending. The intuition for the surprising conclusion is provided clearly in the research: the subsidized SMEs drive non-subsidized bidders to give up more of the gain on the contracts they award. Given the novel features in the empirical methodologies, the surprising results from the simulation study as well as the policy implication from the analysis, this research is highly valued not only in economic literature but also by policy makers in many countries.

Because it naturally would fit better to the small businesses, many of them serve to government contracts as subcontractor. The second essay "Procurement Auctions with Pre-award Subcontract Competitions" theoretically considers pre-award subcontract auctions included in government procurement auctions. In subcontract auctions, businesses submit a bid to prime contractors analyze the procurement auctions including subcontract auctions, this research establishes a two-stage auction model. The analysis proves the existence of a symmetric increasing equilibrium in both upstream and downstream auctions and shows that revenue equivalence holds only in the downstream auction. Then, the following new findings are made: From the viewpoint of the primary contractor's expected profit, first-price auctions weakly dominate second price auctions as a mechanism of subcontractor selection in upstream auctions. Also, optimal reservation price in the downstream auction depends on the number of bidders in the downstream auction. These insights should not only contribute to the theoretical development of auction analysis but also should lead empirical studies of auctions to new lines of research.



Furthermore, in the final essay “Equilibria in Asymmetric Auctions with Entry,” I conduct a theoretical consideration regarding the participation decision of small businesses in the competitive bidding processes. Relying on the model of asymmetric auctions with affiliated private values, I analyze the strategic interaction between large and small businesses on their entry into the procurement auctions. A contribution of this research is to propose the best mechanism which induces the best entry to be able to maximize competition and minimize procurement costs.

the expected relax the symmetric assumption (i.e., the potential bidders may not be ex ante the same). The main findings are threefold. First, auction is optimal (revenue maximizing) if and only if the mechanism is ex post efficient. Second, without any participation control, a coordination problem in which only the lower value bidders participate and the higher value bidders stay out is likely, which makes the auctioneer worse off. Finally, there is an entry fee/subsidization scheme which, together with an ex post efficient mechanism, implements the optimal outcome as a unique equilibrium. Contrary to the existing theorem which claims that in asymmetric auctions well-designed ex post inefficient mechanisms are optimal (e.g., Myerson 1981, McAfee and McMillan (1987)), the results show that, even in an asymmetric auction, the mechanisms with free entry and no distortion are optimal taking into account the potential bidders endogenous participation.

The remainder of this chapter will provide a background of the literature relating to theoretical and empirical models of procurement auctions. Chapter 2 examines the effects of small business set-asides on government procurement costs. Chapter 3 devotes to the theoretical analysis of procurement auctions with pre-award

subcontract auctions. Chapter 5 conducts a theoretical consideration on the strategic interaction between advantaged and disadvantaged potential bidders regarding their participation into an auction game. Chapter 5 summarizes the findings and concludes.

## **1.2 Literature review**

This section provides a review of the literature regarding the research of auctions with entry, empirical analysis on small business programs, and subcontracting in the procurement market.

### **1.2.1 Small business programs**

Ayres and Cramton (1996) investigate the affirmative action program in United States FCC spectrum auctions. Their case studies focus on the “regional narrowband” auctions of thirty licences for use in advanced paging service. In the FCC’s affirmative action, disadvantaged bidders, such as small businesses and woman or minority owned firms, are granted a 40 percent bidding credit on ten of the thirty narrowband licenses, as well as a subsidy for their interest payments. Since the combination effect is that favored bidders had to pay the government only 50 percent of a winning bid, they consider that the credit is large enough to discourage entry by advantaged firms. Surprisingly, their estimation suggests that this effective set-aside program increases the government’s revenues by approximately \$45 million, or 12 percent of the government’s total auction revenue. They also note that set-aside auctions are able to raise the expected auctioneer’s welfare if 1) there is insufficient

competition among strong bidders; 2) the seller is able to identify who is strong or weak; 3) resale is prohibited.

Denes (1997) provides the first thorough analysis for the impact of small business set-asides in public procurement. He investigates the federal dredging contracts during 1990 and 1991 and examines the mean values of set-aside (or restricted) bids compared with the mean values of the related unrestricted bids on the data in eight categories and performs a series of paired *t*-tests. He finds that in all but one instance, there is no significant difference between the bids submitted for set-asides and the bids submitted on unrestricted solicitations and concludes that there is no evidence to suggest that set-asides are costly. According to his study, 3.6 firms bid on the set-asides, whereas only 3.1 firms bid when set-asides were not employed, which, he suggests, induces either no change or a lower bid price on the set-asides.

Marion (2004) recently provided a more systematic analysis for affirmative action in the public procurement. He investigated the effect of the bidding credit program in highway procurement auctions by the California Department of Transportation. Then, he argued that by granting a bid preference to higher-cost bidders, the government loses surplus from lower-cost bidders by awarding contracts to likely higher-cost competitors. At the same time, the preferential treatment increases the competitive pressure exerted by favored bidders. In descriptive regressions, he found that the auctions with bidding credits increase procurement costs by 3.5 percent, possibly because the likelihood of large firm participation is smaller for preference auctions than for non-preference auctions.

### 1.2.2 Subcontracting

There are fairly large volumes of existing literature which deal with procurement auctions with subcontracting (Kamien et al. (1989) and Gale et al. (2000), Marechal and Morand (2003), Wambach (2008)). However, this research made the first attempt to model both upstream and downstream auctions at the same time and examines the mutual influence. A non-trivial extension made in this research from Hansen (1988) is lies in the detailed examination about the optimal mechanism and any other efficiency analysis on the downstream auction. The key assumption to cope with the convoluted random variables is increasing hazard rate (IHR), which is closed under convolution (Barlow and Proschan (1975)).

The model of auctions I use is in fact the closest to Hansen (1988), where he argues that bidders bid more aggressively if there is a downstream market in which the quantity demanded is determined by the winning bid price. A non-trivial extension I make to Hansen lies in modeling the downstream competition with using an auction game. It enables us to provide qualitative examination on the downstream market, such as optimal design and efficiency analysis.

### 1.2.3 Auction with Entry

The theory of optimal design (revenue maximization) has been developed by Riley and Samuelson (1981), Myerson (1981), Bulow and Roberts (1989), McAfee and McMillan (1989)) asserting that a positive reservation price or some distortive allocation favoring a group of bidders improves revenue for the seller. A restriction of the models stems from the assumption that the number of bidders is exogenous.

On the other hand, the theoretical analysis of auctions with endogenous participation consists of two groups of literature. One group investigates either an asymmetric equilibrium (*e.g.*, McAfee and McMillan (1987b)) or a symmetric equilibrium (*e.g.*, Levin and Smith (1994)), assuming that the potential bidders decide whether to enter the auction before acquiring their signal. In contrast, the other group analyzes an entry equilibrium where potential bidders first obtain a signal and then make their entry decision Samuelson (1985).

Finally, the results obtained in this research provide a theoretical background for the experimental analysis for auctions with endogenous participation. Pevnitskaya (2003b) observes an evidence in the laboratory experiments that the same subjects are more likely to participate than the others.

## CHAPTER 2

### SMALL BUSINESS SET-ASIDES IN PROCUREMENT AUCTIONS: AN EMPIRICAL ANALYSIS

#### Abstract

As part of public procurement, many governments adopt small business programs to provide contract opportunities for businesses often with preferences for firms operated by members of groups designated as disadvantaged. The redistribution arising from such programs, however, can introduce significant added costs to government procurement budgets. In this paper, I examine to what extent small business set-asides increase government procurement costs. The estimates employ data on Japanese public construction projects, where approximately half of the procurement budget is set-aside for small and medium enterprises (SMEs). Applying a positive relationship between profitability and firm size obtained by non-parametric estimation of asymmetric first-price auctions with affiliated private values, I conduct a counterfactual simulation to indicate that approximately 40 percent of SMEs would exit from the procurement market were set-asides to be removed. Surprisingly, the resulting lack of competition would actually increase government procurement costs, more than offsetting the production cost inefficiency.

## 2.1 Introduction

As part of public procurement, many governments adopt a program for encouraging small businesses to participate in procurement auctions.<sup>1</sup> In the United States, the Small Business Administration suggests almost all agencies in the federal government spend an overall proportion of 23 percent of their procurement budget with small firms.<sup>2</sup> For some departments, such as the Department of Transportation, the expenditure for small firms in 2005 was approximately \$670 million, which accounted for 45 percent of the total annual expenditure. A similar program is seen in public procurement in Japan. For the central government, the spending target to small and medium-sized enterprises (SMEs)<sup>3</sup> was 50.1 percent in 2007.<sup>4</sup> As in the case of the U.S. federal government, the goal is achieved almost every year.

Reserving a proportion of contracts to small businesses restricts competition, which can result in the market being inefficient and costly. Nevertheless, some theoretical literature of auctions predict that set-asides may not hurt procurement budgets as much as expected. For instance, Ayres and Cramton (1996) investigate the affirmative action program in the FCC spectrum auctions and observe that setting aside some contracts for disadvantaged bidders enhances competition among

<sup>1</sup>Bannock (1981) identifies United States, Germany, Switzerland, and Japan as the countries in which governments strongly support small businesses.

<sup>2</sup>The Federal Acquisition Regulation (FAR), Subpart 19.5. states that if the contracts are no more than \$100,000, these are automatically reserved exclusively for small business concerns and shall be set aside for small business.

<sup>3</sup>SMEs are defined as 1)the firm hires less than three hundred employees or the amount of its capital is equal to or less than a hundred million Yen in Japan. This criteria is applied to the industries of manufacturer, construction, transport, and so on. Service businesses or some other businesses are applied with slightly different criteria.

<sup>4</sup>The law “Ensuring Opportunities for Procurement of Receiving Orders from Government” encourages each ministry to employ set-asides so as to achieve the goal.

advantaged bidders, which can compensate the efficiency loss. Milgrom (2004) points out the analog of set-asides for price discrimination conducted by a multi-market monopoly seller.

Nonetheless, the empirical literature in this field is somewhat lacking. In particular, to the best of my knowledge, there is no existing work which estimates the extent that set-asides hurt government budgets.

This paper makes the first attempt to investigate the effect of set-asides on government budgets by using structural estimation techniques. In particular, I quantify how much government procurement costs are changed by a SME set-aside program, and to what extent SMEs' entry to procurement markets is promoted.

The data I used in this research is from Japanese public procurement auctions for civil engineering works conducted by the Ministry of Land, Infrastructure and Transportation (MLIT), the largest procurement buyer of public works in the country. From April 2005 and March 2008, the ministry spent nearly \$20 billion<sup>5</sup> for approximately 11,000 civil engineering contracts, having accepted nearly 100,000 bids. The ministry set asides approximately two thirds of the procurement budget of civil engineering projects for SMEs.

Another source of data is the government database for certified contractors. It provides the contractors' information about their annual sales, amounts of capital and debt, number of engineers and employees, rate of fatal accidents and so on. Based on the information, I control for firm size in this analysis so as to measure

<sup>5</sup>It is calculated by  $\$1 = \text{¥} 105$ .



the quantitative relationship between firm size and profitability from competitive bidding processes.<sup>6</sup>

To examine the effect of a small business program on procurement costs, one must know what the contract prices would be should the government eliminate the program from the procurement market. However, those data is not available. Therefore, a counterfactual simulation is needed to conduct comparative statics analysis of small business set-asides.

I begin the counterfactual simulation by creating the competition between large firms versus SMEs. Because of set-asides, the number of sample auctions in which large firms and SMEs compete with each other is considerably limited.<sup>7</sup> However, the size of firms participating in each auction differs from one to another. The approach I take in this study is to regress the recovered production costs (and profitabilities) on firm sizes in each sample auction so as to measure the overall quantitative relationship between profitability and firm size in procurement auctions.

Therefore, my empirical analysis consists of the following three-steps. First, I use a procedure of nonparametric estimation for asymmetric first price auctions with affiliated private values (APV) to identify the bidders' costs from observed bids. Then, as a second step, I use regression analysis to find the quantitative relationship between firm size and profitability in procurement auctions, where profitability (expected payoffs) is defined by the profit margin (bid minus cost) times the probability of winning. Finally, I construct a static entry model where I employ the

<sup>6</sup>The central and local governments use the information to assess whether contractors qualify for small businesses.

<sup>7</sup>Although limited, there are auctions in which large firms and SMEs compete with each other since by low set-asides are not permitted in the case where there are too few SMEs to provide sufficient competition.

obtained relationship between expected payoffs and firm size. Regarding the estimated expected profits as a payoff from entry, the entry model predicts how many SMEs would drop out because of large firm entry into a market which was previously reserved exclusively to SMEs under the set-aside program. Furthermore, comparing the winning bid data with respect to the number of participants in each auction, I estimate how much the resulting lack of competition affects government procurement costs.

The model of auctions with entry is based on a two-stage game: potential bidders decide whether to enter the first stage, and the second stage is a first price auction. The first stage relies on the assumption that entry is sequential and the number of firms is treated as a continuous variable. Similar to McAfee and McMillan (1987b), I also suppose that all the actual bidders must incur a fixed cost prior to bidding in order to know their own signal. In this setting, I plug the relevant estimates from the empirical analysis to simulate the case where the set-aside program were to be ineffective. The virtue of the model is that I can separate the bidders' behavior in the auction game from the entry game.

Surprisingly, my estimation results suggest that the program actually saves government procurement costs. Applying the quantitative relationship between firm size and productivity to the average difference in firm size between large firms and SMEs, I find that on average the production cost of SMEs is 1.2 percent higher than that of large firms. Similarly, based on the quantitative relationship between firm size and winning frequency, it is found that an SME would win 5.2 percent less frequently than a large firm if an SME and a large firm competed one-on-one.

These small differences in costs and winning probability lead to a non-trivial difference in profitability between the two groups of bidders. The expected payoff of an SME would be 43 percent lower than that of a large firm when both compete in the same auction. The simulation result indicates that, due to the disadvantage in profitability, SMEs' participation would drop by 38 percent on average were set-asides to be removed. Consequently, the large firms' shifting their entry to originally set-aside projects would cause the following two competing effects on procurement costs. The prices of the originally set-aside projects would fall due to the entry by cost efficient large firms, whereas the prices of the related projects which would have been reserved exclusively to SMEs under the set-aside program would rise because of an approximately 43 percent decline in number of large firms. The simulation studies suggest that the latter effect dominates the former in my simulation so that the program should decrease the procurement costs by 0.28% percent.

The empirical results conclude that the set-aside program has been successful. It improves equity between advantaged and disadvantaged firms without substantial increase of procurement costs. The results not only correspond to the prediction by the theoretical literature on asymmetric auctions, but also are in line with the seminal empirical work of Denes (1997) on set-aside programs, despite the difference in approach and data. In addition, my structural estimation further illustrates that the subsidized SMEs drive non-subsidized bidders to give up more of the gain on the contracts they award. The large firms' expected net gain is almost zero while it would be 1.82 percent of the estimated project cost without the small business program. In other words, set-asides squeeze more rents from large firms, which

enables the procurement buyer to lower procurement costs, more than offsetting the resulting production cost inefficiency.

The remaining part of this paper is organized as follows. Section 2 addresses the related literature. Section 2.2 provides a brief explanation on public construction procurement markets in Japan. Section 2.3 describes the data. A theoretical and model of asymmetric first-price auctions is provided in section 2.4. Section 2.5 is devoted to describe theoretical and empirical models about auctions with endogenous participation. Section 2.6 illustrates the estimation and simulation results. The final section provides further discussion and conclusion. Proofs are given in an appendix.

## **2.2 Public construction markets in Japan**

### **2.2.1 Overview**

Investment in the construction industry accounts for nearly 20 percent of the country's GDP and employs over 10 percent of the working population in Japan. The percentage of public investment as a portion of all construction investment is 45.6 percent in 2001.

Public account law enforces all governments and public entities to use competitive bidding when they acquire any goods and services unless the contract amount is sufficiently small.<sup>8</sup> Three types of bidding systems are used in the public sector: 1) open competitive bidding, 2) invited bidders, and 3) contract at discretion. Although not a majority, scoring tenders are also used in the awarding mechanism, in

<sup>8</sup>Every contract of construction project the amount of which is above 2.5 million Yen and the contract of any other goods and services the amount of which is above 1 million Yen must be procured through auctions.

which bidders submit not only the price but also another variable such as term of work or quality of work.

An idiosyncratic feature of the Japanese public procurement system is in the screening process for bidders. Contractors must take a preliminary qualification exam in order to bid for projects. The exam measures the firm's technological, financial and geographical status and gives them scores as a result of evaluation. For each auction, the procurer selects, or makes an announcement to, a set of legislated contractors as qualified bidders, and the selection is based on the exam results.

In procurement auctions, governments face the risk of awarding the contract to less-qualified or inferior firms, which would later end up defaulting. Some projects demand advanced technologies and skills, as well as a sufficient amount of capital to complete.<sup>9</sup> To mitigate such an asymmetric information problem, screening processes for selecting qualified bidders are essential to the success of the auction.<sup>10</sup> The preliminary qualification exam works in the same manner as the bonding system in the U.S. public construction market. A brief discussion about the preliminary qualification examination in Japan is provided in the next section.

Another major difference in the Japanese procurement system is in the contract principle. Unlike in the United States or many other countries, construction contracts are based on total price contracts, in which bidders submit only a total price without necessarily itemizing unit prices. Instead, engineer's offices regularly update market price lists and uses them in the event that a change order is called for

<sup>9</sup>The possibility of default or non-performance can have perverse effects on the bidding in the auction; a bidder with a high likelihood of default tends to be chosen as a winning bidder. See Zheng (2001) for more detail.

<sup>10</sup>See also Bajari and Tadelis (2001), Laffont and Tirole (1994) for more discussion about the importance of the screening processes in procurement auctions.

during a certain performance. The yearly updates on these price lists are based on hearing investigation, but the survey is conducted independently from procurement auctions. Unfortunately, there is no formal theory which analyzes the effect of contract formats on bidding behaviors. So, my empirical analysis ignores the contract format effect.

Finally, the announcement policy of reservation price and engineer's estimated costs differs from many other countries, in which these are typically opened prior to bidding in auctions. On the other hand, in most public procurement auctions in Japan, such information is secret until the auction is over. However, the secrecy of reservation price is mitigated with the auction design. If no bid is below the reservation price, the next round auction begins immediately with the same member. This process goes on at most three times. The project is reserved unless any contractor bids below the reservation price at the third round. In this sense, reservation prices are almost unbinding in the first round.

### **2.2.2 Preliminary qualification examination**

Preliminary qualification certifies a set of firms as bona fide bidders in procurement auctions to protect the owner of a project against the risk of non-performance. Similar screening processes are widely used at public procurement auctions in European countries and work in the same manner as the bonding system in U.S. public construction auctions in terms of reducing the risk of contractor's default.

The preliminary qualification in Japan is based on the firm's disclosure of information with respect to their financial and technological performance. In particular, information includes annual sales, number of engineers in each area of expertise,

experience and business history and so on. Based on the set of information as well as the evaluation of work performed, governments measure the firm's overall ability to perform. As a result of the exam, the qualified firms typically obtain two kinds of scores for each area of their expertise.

The first score is called the "Business Evaluation" (BE) score, which is essentially a weighted average of 1) annual value of completed construction works by license classification, 2) number of technical staff, 3) business conditions (based on financial statement analysis), 4) number of engineers, 5) record of safety performance. For the qualified bidders of MLIT, the maximum and minimum scores are 1859 and 329 with an average of 851.1. The detailed summary statistics on Business Evaluation are available in section 2.3. The BE score is given through the countrywide criteria of measurement specified in construction industry law, so that each firm has a unique score value for each expertise.<sup>11</sup>

On the other hand, the second score, which is called the "Technology Evaluation" (TE) score is the past performance evaluation measured by each procurement buyer.<sup>12</sup> Unlike BE score, the measurement criteria varies across procurers. Hence, BE and TE scores may not both be high. For instance, if a firm performs very well in the projects of a particular procurement buyer, the TE score of the procurer should be high, but BE could be low.<sup>13</sup> If a government has multiple local divisions, each may have a different evaluation criteria for the TE score.

<sup>11</sup>The number of expertise is 28, which is specified in the construction industry law. Firms must obtain a licence for each area of expertise to operate.

<sup>12</sup>The criteria typically reflects the firm's past works such as contribution to the quality of projects, schedule of works.

<sup>13</sup>Business Evaluation is often called "objective" due to the fact that it reflects the firms' absolute ability to work, whereas the technology score is called "subjective" because it represents how a particular procurer evaluates the performance of the firm.

The assessment on whether a firm is favored in the set-aside program is based on the sum of the BE and TE scores. However, the business evaluation and the total score are strongly correlated with each other.<sup>14</sup> To avoid the heterogeneity of TE across locations, my analysis only uses the BE scores as the control variable for the corporation size.

### 2.2.3 Set-asides in the public construction market

The selection rule for bidders is primarily based on “size matching rule.” When a particular project is auctioned, a set of bidders are chosen so that their sizes will match the project size. For instance, only large firms are qualified to participate in the auctions for large and high-end projects and are not allowed to bid in small and low-end projects, which are reserved to SMEs.<sup>15</sup> The size matching rule has a priority in the selection of bidders unless the number of designated bidders is too small to provide adequate competition.

Set-asides are the only explicit method in favoring SMEs in Japanese public procurement auctions. Every year, Japanese central and local governments determine the objective set-aside budgets by which the governments should assign contracts

<sup>14</sup>More precisely, governments assign grade for each firm based on the total score. For instance, MLIT gives either “A”, “B”, “C”, or “D” for each certified contractor with the civil engineering expertise, where A is top grade. Large contractors are likely to have “B” or higher, and is likely to have “A” grade if the firm is countrywide operated. Based on the grade, governments implement the set-aside program in such a way that the firms with A or B grade are excluded to bid for low-end projects.

<sup>15</sup>Set-asides are implemented as part of the size matching rule. In the case of MLIT, it also grades every civil engineering works from A to D according to the size, where grade A implies the highest-end. Engineer’s estimated costs are typically used as a proxy to determine the project size. Under the size matching rule, contractors are selected, or allowed to participate in the auction so that their grades match the project grade.



to SMEs.<sup>16</sup> In 2005, central governments and public entities spent ¥8.8 trillion to purchase land and items, construction works, and services. ¥4.1 trillion was expended to SMEs, which accounted for 46.9 percent of the total budget (The target amount was ¥4.3 trillion, accounting for 46.7 percent). For the Ministry of Land, Infrastructure and Transportation, 50.8 percent of entire expenses was allocated to SMEs in the year. To achieve the goal, approximately two thirds of civil engineering contracts were set-aside for SMEs.

## 2.3 The data

### 2.3.1 Overview

The data I use in the analysis contains the bid results of the procurement auctions for civil engineering projects from April 2005 through March 2008. The number of contracts awarded was 11,114 during this period.

MLIT posts the bid results on the website, Public works Procurement Information service (PPI).<sup>17</sup> The information available in PPI includes the name of orderers (local branch name), project names, project types, date of auctions, reservation prices, auction formats (open competitive bidding or invited bidders) and submitted bids with the bidder's name.<sup>18</sup> PPI also provides the lists of all the qualified firms, which consist of the address of the firm's headquarters, the name of owner,

<sup>16</sup>This policy is specified by the law "Law on ensuring the receipt of orders from small and medium enterprises."

<sup>17</sup>The address is "http://www.ppi.go.jp."

<sup>18</sup>The information about work location is not available in general.

business and technology scores as well as total scores and grades for each area of expertise. All the data in this empirical study is from the website.

MLIT procures 21 types of construction works including civil engineering (or heavy and general construction works), buildings, bridges, paving, dredging, and painting. The amount of civil engineering projects is approximately ¥ 750 billion a year, which accounts for approximately 54 percent of the entire expenditure of the ministry as indicated in figure 2.1 and 2.2 as well as for 7 percent of the public construction investment in the country.

MLIT has 9 regional development divisions in 9 regional districts. The data includes the civil engineering projects in 8 districts indicated in figure 2.1. Each of the regional development divisions has a certified firms' list from which it chooses the bidders for each procurement auction. The lists are updated every two years. The total number of firms on the lists was 43,522 in April 2007. Since large firms typically operate across several regions, it is often the case that a particular firm is listed on two or more of these lists. The number of firms without such duplication is 32,993, which accounts for approximately 20 percent of all the licensed civil engineering construction firms in Japan.<sup>19</sup>

The data has some limitation in identification of contractors. The bid results provides the bidder's company name only. So, in the case that two or more different firms have an identical company name, the bidder must be inferred.<sup>20</sup> So I narrow the candidate list down by whether, i) location (prefecture) of the project matches the location of headquarters, and ii) bidder's size matches the project size according

<sup>19</sup>The total number of licensed civil engineering firms is 167,896 in 2005 (MLIT, 2005).

<sup>20</sup>For example, there are seven "Showa Kensetsu Co.ltd" on the contractor list of Kanto district Development Bureau. The bid results do not indicate which "Showa Kensetsu" actually bid.

Category	2007			2006			2005		
	Amount	(%)	Count	Amount	(%)	Count	Amount	(%)	Count
General & Heavy	806,667	(54.3)	4,092	606,795	(47.9)	4068	626,493	(53.3)	3,752
Paving	113,252	(7.6)	789	132,088	(10.4)	862	123,043	(10.5)	799
Bridge	118,931	(8.0)	218	96,592	(7.6)	208	30,407	(2.6)	148
landscaping	10,858	(0.7)	313	10,301	(0.8)	311	13,524	(1.2)	356
Architecture	60,817	(4.0)	674	51,404	(4.1)	692	42,670	(3.6)	717
Painting	6,552	(0.4)	223	6,800	(0.5)	231	6,009	(0.5)	241
Maintenance	151,119	(10.2)	2,535	141,751	(11.2)	2718	138,855	(11.8)	2,919
Dredging	5,929	(0.4)	25	4,792	(0.4)	19	4,050	(0.3)	21
Machinery & equipment	27,355	(1.8)	434	37,102	(2.9)	560	37,221	(3.2)	594
Info & telecom facility	43,824	(3.0)	731	37,102	(2.9)	848	43,657	(3.7)	998
Others	139,645	(9.4)	1,209	140,766	(11.1)	1,216	108,786	(9.3)	1,118
Total	1,484,949	(100.0)	11,243	1,265,492	(100.0)	11733	1,174,715	(100.0)	11,663

Table 2.1: Projects Yearly (¥million)

Category	FY2005-2007		
	Amount	(%)	Count
General & Heavy	2,039,956	(52.0)	11,912
Paving	368,383	(9.4)	2,450
Bridge	245,930	(6.3)	574
landscaping	34,683	(0.9)	980
Architecture	154,891	(3.9)	2,083
Painting	19,361	(0.5)	695
Maintenance	431,726	(11.0)	8,172
Dredging	1,477	(0.4)	65
Machinery & equipment	101,678	(2.6)	1,588
Info & telecom facility	124,582	(3.2)	2,577
Others	389,198	(9.9)	3,543
Total	3,925,156	(100.0)	34,639

Table 2.2: Projects from FY2005 through FY2007 (¥million)

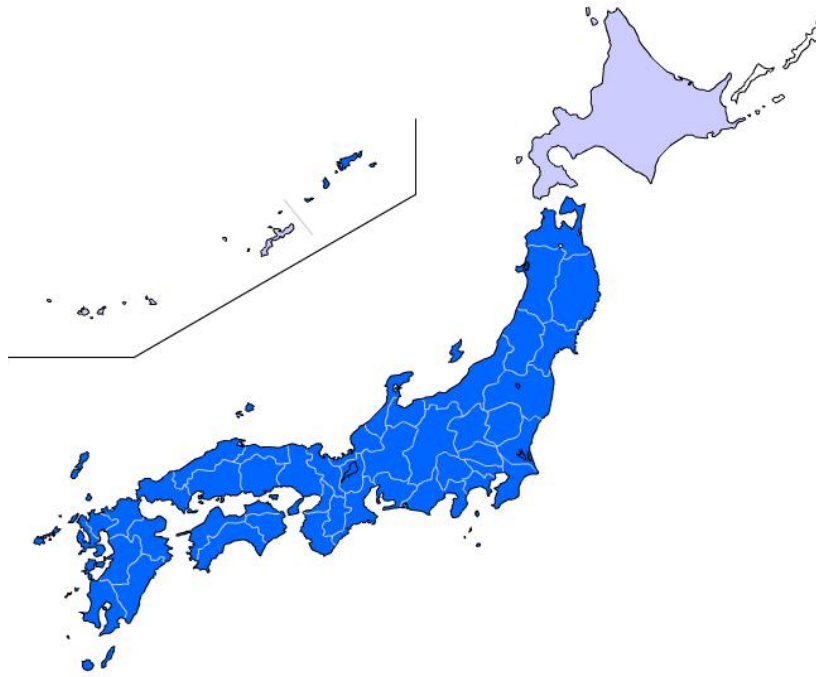


Figure 2.1: Data area

to the size matching rule. Through this process, I am able to identify almost all contractors on the bid results. I assume the unidentified firms to be the average sized firm in the auction.

## 2.3.2 Summary statistics of bids and scores

### Normalization of bidder's size

In the observations, each auction has a unique set of bidders in general. Hence, a firm with a higher score can be a smaller bidder if the opponents have a much higher score, and vice versa. To model the firm's size in comparison to the size of its opponents, I normalize the Business Evaluation score (hereafter, normalized score) as the following procedure.

Suppose there are  $m$  auctions and the number of bidders in auction  $k = 1, \dots, m$  is denoted by  $n_k$ . Let  $X_{i,k}$  be the value of the Business Evaluation score of the  $i$ th lowest bidder in auction  $k$ .<sup>21</sup>

Each bidder is not informed of the competitor bidders. In fact, each bidder has some information about who the opponent bidders are, guessing through the project location, project size, the competitor's backlog and so forth. Hence, my stylized model assumes that the  $i$ th bidder in the  $k$ th project knows the average of opponent bidder  $\bar{X}_{-i,k} = \sum_{j \neq i} X_{j,k} / (n - 1)$ , but not for each  $X_{j,k}$ .

<sup>21</sup>Note that the analysis assumes that the bidders' asymmetry comes from their size, all the information with respect to the bidder's identity in the observation is discarded except the relative size.

The mean bid  $\bar{X}_k = \sum_{\tau=1}^n X_{\tau,k}/n$  in the  $k$ th auction is known to each bidder.

The normalized score is then calculated as,

$$x_{i,k} = \frac{X_{i,k} - \bar{X}_k}{\bar{X}_k}.$$

Because of the assumption for  $\bar{X}_{-i,k}$ , the value  $x_{i,k}$  not only represents the relative size of the  $i$ th bidder in auction  $k$ , but also informs the  $i$ th bidder about the average relative size of his opponents. For instance,  $E[x_{j,k}|x_{i,k}]$  will be negative for any  $j$  if and only if  $x_{i,k}$  is positive.<sup>22</sup>

	No. Obs.	Mean	Std.Dev.	CV*	Max	Min
BE Score : $X_{i,k}$	100,585	1017.176	133.58	0.132	1,859	506
Normalized Score : $x_{i,k}$	100,438	-.001	.074	-	0.50	-.423

\* Coefficient of variation

Table 2.3: Summary statistics : The business evaluation score of actual bidders

Table 2.3 provides the summary statistics on  $X_{i,k}$  and  $x_{i,k}$  of the actual bidders. Figure 2.2 depicts the histogram for  $x$ , the normalized score. The effect of the set-aside program is glimpsed from the fact that the coefficient of variation (CV) on  $X_{i,k}$ , which is defined by the standard deviation divided by the mean of  $X_{i,k}$ , is approximately 13 percent. So if bidders are randomly picked in each auction, the standard deviation of  $x_{i,t}$  would be 13 percent. However, the actual standard deviation is 7.6 percent, implying that the participation restriction by government reduces the asymmetry of bidders.

<sup>22</sup>It is because  $\sum_{\tau \neq i} x_{\tau,k} + x_{i,k} = 0$  in my model.

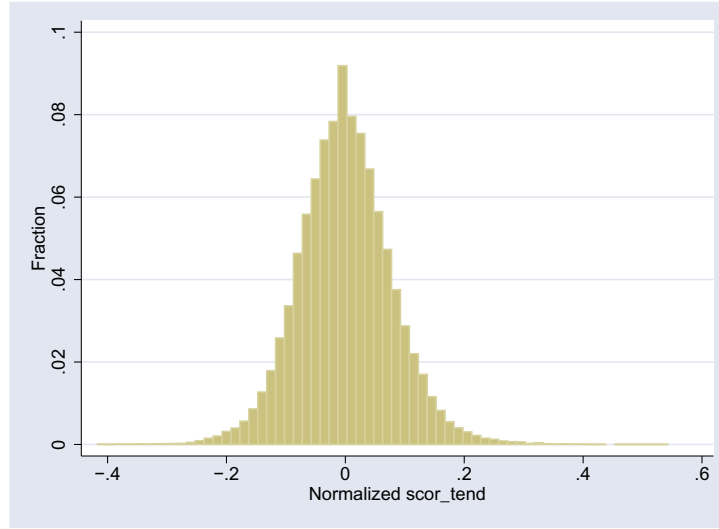


Figure 2.2: Normalized score of actual bidders

### Percentage bids

Figure 2.3 describes the histogram on the project size. Since each construction project is unique, there still remains a great deal of heterogeneity in project size. The most typical contract amount is approximately ¥100 million measured in the engineer's estimated costs. The largest is approximately ¥12 billion, while the smallest is less than ¥1 million. Table 2.4 breaks down the summary statistics of project size.

To eliminate the project heterogeneity, all bids in the empirical analysis are described by the percentage with respect to the engineer's estimated cost. If the  $k$ th auction is the price only auction, then the percentage bid of the  $i$ th lowest bid

Project Size	No. Observation	Engineer's Estimate Costs			
		Mean	Std.Dev.	Max	Min
730 or more	228	1,974.13	1,458.72	10,490.00	737.90
300 - 730	521	469.09	115.00	717.00	300.10
60 - 300	8,851	141.55	63.23	300.00	60.01
60 or less	1,514	37.69	17.01	59.99	.01
Total	11,114	180.35	348.14	10,490.00	.01

¥ Million\*

\*Money amount is based on the engineer's estimate.

Table 2.4: Summary Statistic on Project Size

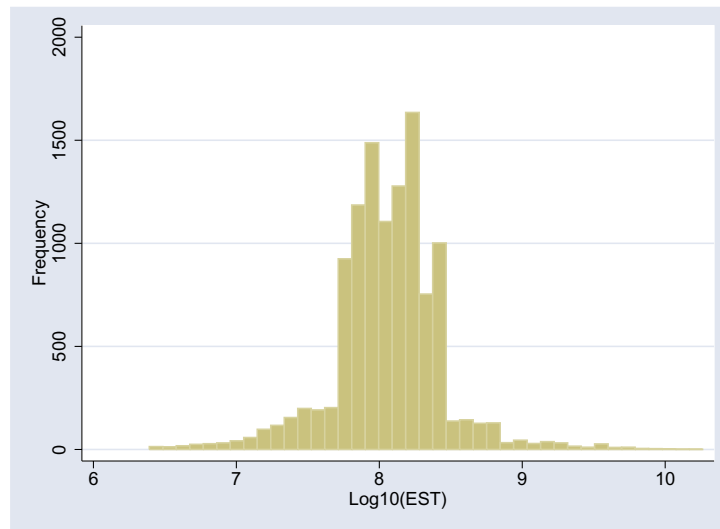


Figure 2.3: Project Size ( $\log_{10}$  of engineer's estimate)



is given by

$$\frac{\text{Bid}_{i,k}}{\text{Est}_k}, \quad (2.1)$$

where  $\text{Bid}_{i,k}$  is the value of the  $i$ th lowest bid and  $\text{Est}_k$  is the engineer's estimated cost for auction  $k$ . The engineer's estimated costs are equal to the reservation price in the procurement auctions by MLIT. If the  $k$ th auction is a scoring auction in which bidders submit not only the price bid, but also some other factors such as quality, completion time and so forth, then the bidder with the highest score wins the project. Therefore, percentage bids for scoring auction are defined by  $\text{Score-Bid}_{i,k}/\text{Reservation-Score}_k$ .

### Regression results for bids on corporate size

It is observed that, in each auction, larger firms bid lower prices than smaller firms. Table 2.5 describes the result of regression for the percentage bids on normalized scores. Auction specific effects are taken into account by fixed effect and random effect models. After dropping 306 out of 11,375 auctions, which contains "throw-away bids" *i.e.*, larger than 200 percent of the engineer's estimated cost, the negative relationship between the normalized bids and size is significant ( $t$ -value : 6.03 in FE estimation).<sup>23</sup> The number of observations after exclusion equals 86,798. Figure 2.4 depicts the bid density of larger firms (The score is 10 % greater than the average) is shifted downward when compared to that of smaller firms (The score

<sup>23</sup>The exclusion of extremely high bids was also done in Corns and Schotter (1999). They mentioned that these observations have to be removed from the sample because of the influence they would have on the estimation.

is 10 percent smaller than the average). Table 2.5 indicates that the bidder's size yields a small but statistically significant difference in bids.

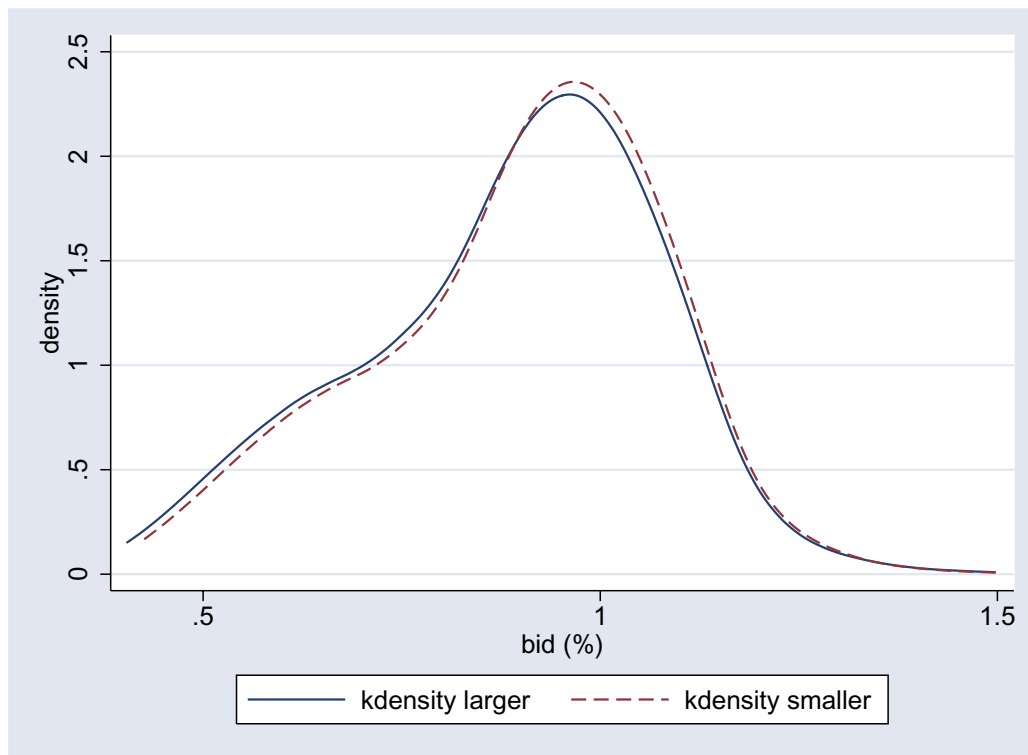


Figure 2.4: Densities (Percentage bids)

Finally, I explore the production capacity utilization in procurement auctions. Both figures 2.5 and 2.6 illustrate that many small businesses on the lists have little chance to bid in spite of the set-aside program. Figure 2.5 depicts the density on Business Evaluation score of the bidders who actually bid, while figure 2.6 shows the score of all the firms on the certified contractor lists. The density shifting toward the left in figure 2.6 indicates that despite of the small business set-asides, a sufficient volume of production capacity remains available in small businesses.

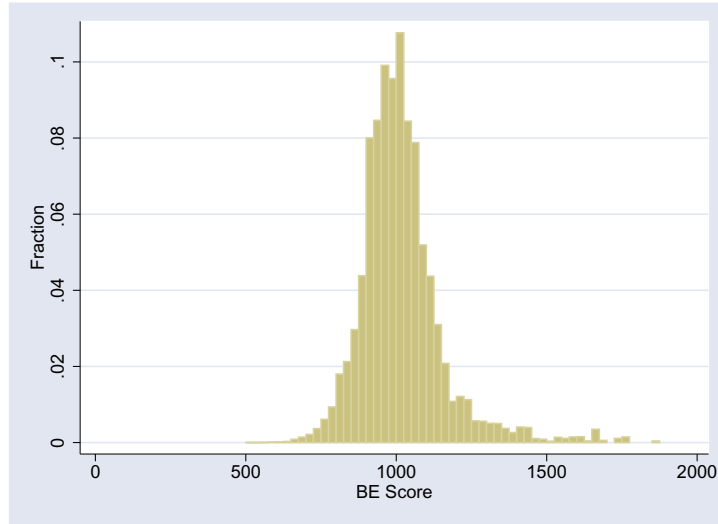


Figure 2.5: BE score of actual bidders

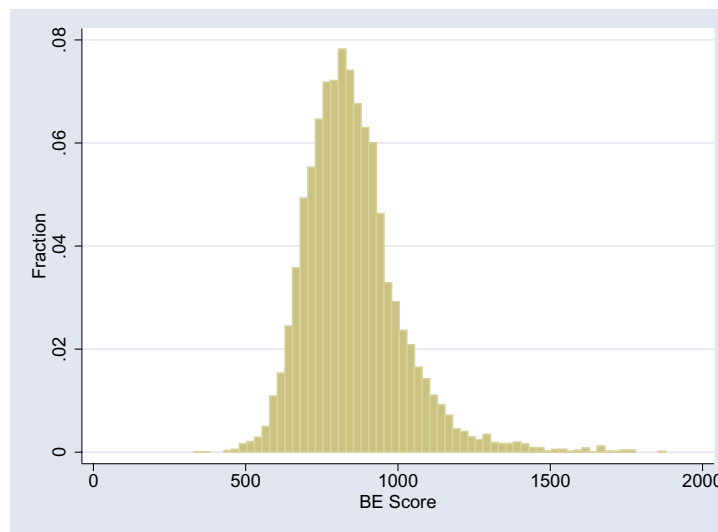


Figure 2.6: BE score of all the firms on the certified contractor lists

	Bids			Costs		
	OLS	FE	RE	OLS	FE	RE
$x_{i,k}$	-0.030 (4.66)**	-0.024 (6.03)**	-0.024 (6.15)**	-0.038 (5.52)**	-0.030 (6.99)**	-0.031 (7.12)**
Auction date	(88.95)** 0.014	-	-	(90.76)** 0.012	-	-
Scoring auction dummy	(4.66)** -0.063	-	-	(5.52)** -0.079	-	-
Auction form dummy 2	(88.95)** 0.014	-	-	(90.76)** 0.012	-	-
Auction form dummy 3	(4.69)** 0.043	-	-	(3.72)** 0.039	-	-
Auction form dummy 4	(21.80)** 0.013	-	-	(21.80)** 0.013	-	-
Constant	(89.55)** 86751	(2951.12)** 86751	(579.76)** 86751	(91.29)** 86751	(2538.72)** 86751	(489.66)** 86751
Observations	86751	86751	86751	86751	86751	86751
R-squared	0.31	0.00	-	0.33	0.00	-
Number of Auctions	-	11058	11058	-	11058	11058

Absolute value of t statistics in parentheses; \* significant at 5%; \*\* significant at 1%  
 $x_{i,k}$ : % difference from average bidders in the  $k$ th auction

Table 2.5: Regression results of normalized bids and estimated costs

## 2.4 Recovery of the bidders' cost distribution

### 2.4.1 Overview

My nonparametric estimation of first price sealed bid auctions is based on Campo et al. (2003), which is an extension of Guerre et al. (2000) to cases with asymmetric bidders with the APV model. The approach of Campo et al. (2003) relies on the assumption that the bidder's asymmetry is represented by a finite number of segments. Hence, if the number of segments is equal to  $d$ , a  $(d + 1)$ -dimensional kernel estimation is required. Therefore, if an empirical model assumes that the bidder's asymmetry is attributed to a continuous variable, then kernel estimation cannot hold.

More recently, Zhang and Guler (2005) proposed a simplified approach by which one only needs a two-dimensional kernel estimation regardless of the structure of bidder asymmetries. The essence of their approach is to estimate the bidder's signal separately for each bidder, expressing each bidder's payoff function in terms of equilibrium distribution of rival bids. They claim that one can avoid suffering from the dimensionality of kernels as long as the set of bidders in the sample is identical. Unfortunately, their approach causes another problem if the data involves the heterogeneity in the set of participants across auctions as it does in this case.

Hence, I reconstruct a model of asymmetric auctions to utilize more samples in kernel estimation. In particular, I assume that each bidder knows its own strength (normalized score) but has limited information about its competitors'. As shown in the next subsection, the bidders are still *ex ante* asymmetric on this assumption. Furthermore, this assumption is more realistic in the actual procurement auctions in which the participants are endogenously determined and nobody knows who the actual opponents are upon bidding.

### 2.4.2 A model of asymmetric auctions

A single and indivisible project is auctioned to  $n$  risk neutral bidders. There is an  $n$ -dimensional distribution with a cumulative distribution function  $H(\cdot)$ . The vector of each bidder's normalized score  $x \equiv (x_1, \dots, x_n)$  is a realization of a random vector with a joint distribution  $H(\cdot)$ . Suppose  $H(\cdot)$ , and  $n$  are common knowledge. Then, for each  $i \in \mathcal{N} \equiv \{1, \dots, n\}$ , the conditional distribution of  $x_{-i} \equiv (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$  and its density are denoted by  $H_{-x_i|x_i}(x_{-i}|x_i)$  and

$h_{-x_i|x_i}(x_{-i}|x_i)$ , respectively. Suppose for all  $i$ ,  $H_{-x_i|x_i}(x_{-i}|x_i)$  has support  $[\underline{x}, \bar{x}]^{n-1}$  and that the probability density function  $h_{-x_i|x_i}(x_{-i}|x_i)$  is continuous in  $x_{-i}$ .

The asymmetric APV model with risk neutral bidders is defined by an  $n$ -dimensional distribution with cumulative distribution function  $F(\cdot|x)$ . The vector of private information  $(c_1, \dots, c_n)$  is a realization of a random vector with joint distribution  $F(\cdot|x)$ . The asymmetry of bidders is captured by  $x$  such that  $x_i$  affects the marginal distribution of  $c_i$  but not the distribution of  $c_j$  for any  $j \in \mathcal{N} \setminus \{i\}$ . That is, the marginal distribution of  $c_i$  is represented by  $F_{c_i}(x_i)$  for all  $i \in \mathcal{N}$ . The affiliation is captured as follows: suppose the  $i$ th bidder's signal is  $c_i$ , then for some  $j \in \mathcal{N} \setminus \{i\}$ , the marginal distribution of  $c_j$  and its density are given by  $F_{c_j|c_i}(c_j|c_i, x_j)$ , and  $f_{c_j|c_i}(c_j|c_i, x_j)$ .

Denote by  $b_i = \beta(c_i|x_i)$  and  $\theta(b_i|x_i) = \beta^{-1}(b_i|x_i)$  the equilibrium bidding strategy, and its inverse, respectively. In equilibrium, the joint distribution of valuations  $F(\cdot|x)$ , and the distribution of bids  $G(\cdot|x)$  are related with  $G(b_1, \dots, b_n|x) = F(\theta(b_1|x_1), \dots, \theta(b_n|x_n)|x)$ . Suppose the marginal distribution of costs  $F_{c_j|c_i}(c_j|c_i, x_j)$  has support  $[\underline{c}, \bar{c}]$  for any  $i \in \mathcal{N}$  and  $j \in \mathcal{N} \setminus \{i\}$ , and that the probability density function  $f_{c_j|c_i}(c_j|c_i, x_j)$  is continuously differentiable (in  $c_j$ ). I also assume that for all  $i \neq j$ ,  $f_{c_j|c_i}(\cdot|c_i, x_j)$  is bounded away from zero on  $[\underline{c}, \bar{c}]$ .

Firm  $i$ 's conditional payoff can thus be written as

$$\pi(b_i|c_i, x_i) = \max_{b_i} (b_i - c_i) \Pr\{b_i \leq B_i|c_i, x_i\},$$

where  $B_i$  is bidder  $i$ 's minimum rival bid, defined as  $B_i \equiv \min\{b_1, \dots, b_{i-1}, b_{i+1}, \dots, b_n\}$ .

Then, I consider an increasing Bayesian-Nash equilibrium in pure strategies. An equilibrium in pure strategies is an  $n$ -dimensional strategy profile  $(\beta(\cdot|x_1), \dots, \beta(\cdot|x_n))$  such that  $\beta(\cdot)$  maximizes  $\pi(b_i|c_i, x_i)$  in  $b_i$  for all  $i$ , and  $c_i$  in its support.

Suppose there exists an increasing equilibrium such that each firm  $i$  bids according to a strictly increasing function  $\beta(c_i|x_i)$ . Then, I define for any  $i \in \mathcal{N}$  and  $j \in \mathcal{N} \setminus \{i\}$  that  $G_{b_j|b_i}(b|b_i, x_i, x_j) \equiv F_{c_j|c_i}(\theta(b|x_j)|\theta(b|x_i), x_j)$  as the probability with which  $b_j \geq b$  for some  $b$ . Note that  $G(\cdot)_{b_j|b_i}$  has the property of probability distribution since  $\beta(\cdot)$  is strictly increasing.

For the  $i$ th bidder, the minimum rival bid  $B_i$  is a random variable conditional on  $b_i$  and  $x_i$ . Therefore, I denote by  $G_{B_i|b_i}(B_i|b_i, x_i)$  the conditional cumulative distribution of  $B_i$ .<sup>24</sup> Then, the bidder  $i$ 's winning probability  $1 - G_{B_i|b_i}(\cdot)$  is given by

$$1 - G_{B_i|b_i}(b_i|b_i, x_i, x_{-i}) = \prod_{j \in \mathcal{N} \setminus \{i\}} \{1 - G_{b_j|b_i}(b_i|b_i, x_i, x_j)\},$$

given that other bidders follow  $\beta(\cdot)$ . Note that  $G_{B_i|b_i}$  is strictly increasing for all  $i$ . Since the bidder  $i$  does not know  $x_{-i}$ , the bidder  $i$ 's *expected* winning probability,  $1 - \bar{G}_{B_i|b_i}(\cdot)$ , is thus given by:<sup>25</sup>

$$1 - \bar{G}_{B_i|b_i}(b_i|b_i, x_i) = \int_{x_{-i}} [1 - G_{B_i|b_i}(b_i|b_i, x_i, x_{-i})] h_{x_{-i}|x_i}(x_{-i}|x_i) dx_{-i}.$$

<sup>24</sup>By affiliation among  $c$ ,  $b_i$  influences  $G_{B_i|b_i}$ , while by asymmetry distribution of  $c$ ,  $x_i$  affects  $G_{B_i|b_i}$ .

<sup>25</sup>The right hand side is more formally expressed as

$$\int_{x_1} \dots \int_{x_{i-1}} \int_{x_{i+1}} \dots \int_{x_n} [1 - G_{B_i|b_i}(b_i|b_i, x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)] h_{x_{-i}|x_i}(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n|x_i) dx_1 \dots dx_{i-1} dx_{i+1} \dots dx_n.$$

Then, the  $i$ th bidder's maximization problem becomes

$$\pi(b_i|c_i, x_i) = \max_{b_i} (b_i - c_i)[1 - \bar{G}_{B_i|b_i}(b_i|b_i, x_i)],$$

given that other bidders follow  $\beta(\cdot|x_j)$ . Then, the  $i$ th bidder's first order condition gives

$$c_i = b_i - \frac{1 - \bar{G}_{B_i|b_i}(b_i|b_i, x_i)}{\bar{g}_{B_i|b_i}(b_i|b_i, x_i)}, \quad (2.2)$$

where  $\bar{g}_{B_i|b_i}(\cdot)$  is the density of  $\bar{G}_{B_i|b_i}(\cdot)$ .

The right hand side of (2.2) gives a unique inverse bid function  $\theta(b_i|x_i)$ . It implies that  $i$ 's strategy is also represented by  $\beta(b_i|x_i)$ . Hence, it is a Bayesian-Nash equilibrium in asymmetric first price auctions with APV. The bidding function can be obtained by solving the system of differential equation represented by  $\theta(b_i|x_i)$  for all  $i$ .

### 2.4.3 Nonparametric estimation

Campo et al. (2003) show that one can estimate the latent value  $c_i$  by using the inverse bid function  $\theta(\cdot)$ . They show that the estimator for costs can be obtained by computing the bid distribution  $\bar{G}_{B_i|b_i}$  and its density  $\bar{g}_{B_i|b_i}$  without solving the system of differential equations.

As in Zhang and Guler (2005), the first step is to interpret (2.2). By definition,  $1 - G_{B_i|b_i}(b_i|b_i, x_i)$  is the probability that the minimum rival bid  $B_i$  is greater than  $b_i$  conditional on  $b_i$ . Also,  $\bar{g}_{B_i|b_i}(b_i|b_i, x_i)$  is the derivative of  $\bar{G}_{B_i|b_i}(\cdot)$ . Hence, (2.2)



can be rewritten as

$$c_i = b_i - \frac{\Pr(b_i < B_i | b_i, x_i)}{\Pr(b_i = B_i | b_i, x_i)}.$$

For estimation, suppose there are  $k = 1, \dots, m$  auctions and that  $n$  bidders bid in each. Then, let  $B_{i,k} = \min_{j \neq i} b_{j,k}$  denote the  $i$ 's minimum rival bid for any sample auction  $k$ . Unlike the standard estimation model, I relax the assumption that the set of bidders in each sample is the same. That is, the  $i$ th bidder in the  $k$ th auction can be different from the  $i$ th bidder in the  $k'$  auction. Thus, the number of combinations of  $x_k \equiv (x_{1,k}, \dots, x_{n,k})$  in the observations is infinitely large.

However, I can easily cope with the problem using the fact that  $\bar{G}_{B_i|b_i}$  and  $\bar{g}_{B_i|b_i}$  depends only on  $x_i$ ; To know the latent value of the  $i$ th bidder in the  $k$ th auction, one can utilize the values of the  $j$ th bid in the  $k'$ th auction as long as the counterpart bidder's score  $x_{j,k'}$  is the same or close enough to  $x_{i,k}$ .

The numerator and denominator in the ratio of inverse bid functions are thus given by

$$\begin{cases} \Pr(b \leq B | b_{i,k}, x_{i,k}) = \frac{1}{mh_G h_x} \sum_{l=1}^m \sum_{\tau=1}^n \mathbf{1}(b \leq B_{\tau,l}) K_G \left( \frac{b - b_{\tau,l}}{h_G}, \frac{x_{i,k} - x_{\tau,l}}{h_x} \right), \\ \Pr(b = B | b_{i,k}, x_{i,k}) = \frac{1}{mh_g^2 h_x} \sum_{l=1}^m \sum_{\tau=1}^n K_g \left( \frac{b - B_{\tau,l}}{h_g}, \frac{b - b_{\tau,l}}{h_g}, \frac{x_{i,k} - x_{\tau,l}}{h_x} \right). \end{cases} \quad (2.3)$$

These hold to the extent that the number of bidders is identical in the sample and there is no heterogeneity in characteristics of projects. In fact, the observations in the paper involve significant heterogeneity in the number of bidders and the characteristics such as locations, project sizes, and auction dates. The next subsection explains how to control for heterogeneity.

## Heterogeneity

I essentially follow Guerre et al. (2000) to control the heterogeneity in the number of bidders and characteristics of each auction. Guerre et al. (2000) address that these are tractable in nonparametric identification by introducing additional dimensions on kernels. The data taken here involve considerable heterogeneity in both the number of bidders<sup>26</sup> and the auction format (menu auctions or price only auctions). The procedure is described as follows.

Let  $z_k$  denote the vector of associated characteristics in project  $k$ . Suppose the bidders' cost distribution for the  $k$ th auction is given by the conditional distribution  $F(\cdot|z_k)$  for some  $z_k$ . Then, the distribution of observed bids in auction  $k$  is given by  $G(\cdot|n_k, z_k)$ . Hence, (2.2) is rewritten as

$$c_{i,k} = b_{i,k} - \frac{1 - \bar{G}_{B_{i,k}|b_{i,k}}(b_{i,k}|b_{i,k}, x_{i,k}, n_k, z_k)}{\bar{g}_{B_{i,k}|b_{i,k}}(b_{i,k}|b_{i,k}, x_{i,k}, n_k, z_k)}. \quad (2.4)$$

Hence, (2.3) becomes

$$\left\{ \begin{array}{l} 1 - \bar{G}_{B_{i,k}|b_{i,k}}(b|b_{i,k}, x_{i,k}, z_k, n_k) \\ = \frac{1}{mh_G^{i,k} h_x^{i,k} h_n^k h_z^k} \sum_{l=1}^m \frac{1}{n_l} \sum_{\tau=1}^{n_l} \mathbf{1}(b \leq B_{\tau,l}) K_G \left( \frac{b - b_{\tau,l}}{h_G^{i,k}}, \frac{x_{i,k} - x_{\tau,l}}{h_x^{i,k}}, \frac{n_k - n_l}{h_n^k}, \frac{z_k - z_l}{h_z^k} \right), \\ \bar{g}_{B_{i,k}|b_{i,k}}(b|b_{i,k}, x_{i,k}, z_k, n_k) \\ = \frac{1}{m(h_g^{i,k})^2 h_x^{i,k} h_n^k h_z^k} \sum_{l=1}^m \frac{1}{n_l} \sum_{\tau=1}^{n_l} K_g \left( \frac{b - B_{\tau,l}}{h_g^{i,k}}, \frac{b - b_{\tau,l}}{h_g^{i,k}}, \frac{x_{i,k} - x_{\tau,l}}{h_x^{i,k}}, \frac{n_k - n_l}{h_n^k}, \frac{z_k - z_l}{h_z^k} \right), \end{array} \right. \quad (2.5)$$

where  $K_G$  is a four-dimensional kernel, and  $K_g$  is a five-dimensional kernel. The regularity assumption for  $F$  and  $G$  is provided in Guerre et al. (2000).

<sup>26</sup>The smallest number is two and the largest 53.

Since my model relaxes the assumption that each auction must have the same sample bidders, the  $h_g$  and  $h_G$  are essentially different for each bidder in the different auction. As usual, the bandwidth is given by  $h_g = c_g(\sum_{k=1}^m n_l)^{-1/6}$  and  $h_G = c_G(\sum_{k=1}^m n_l)^{-1/5}$ , where  $c_G = c_g = 2.978 \times 1.06\hat{\sigma}_b$  by the so-called rule of thumb. I employ the following triweight kernel in the nonparametric identification:

$$K(u) = \frac{35}{32}(1 - u^2)^3 \mathbf{1}(|u| < 1).$$

I execute the calculation using a program written in C++ and takes approximately an hour to obtain a hundred thousand latent variables.

The informational rent decreases as the number of bidders increases. Figure 2.7 shows the bidding function in the case of a small number of participants (5 bidders), and figure 2.8 describes the case of many participants (between 22 and 28 bidders). In both figures, the dark plots represent the bidding function and the light plots represent the forty five degree line. The bid margins are larger in the case of a smaller number of competitors.

Table 2.5 shows the regression result for the estimated costs as a function of firms' size. Again, both the fixed and random effects control for the auction specific heterogeneity, and all the throw-away bids (greater than 200 percent of reservation price) are dropped in the regression. Table 2.5 suggests statistical significance (t-value : 6.99 in FE regression) that large firms have a cost advantage.

Literature on asymmetric first-price auctions predict that disadvantaged bidders bid more aggressively than advantaged bidders in an auction. Table 2.6 shows the regression result of a log bid margin (a submitted bid minus the estimated cost)

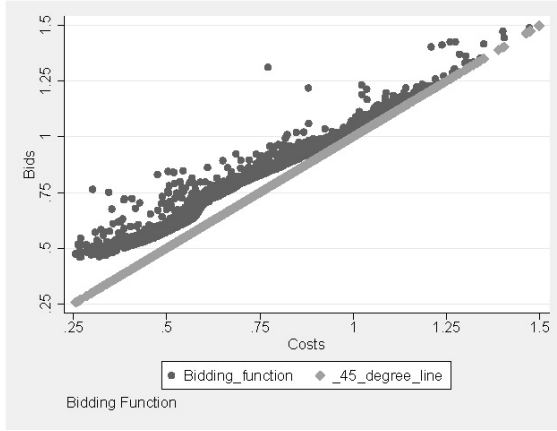


Figure 2.7: Few bidders:  $n = 5$

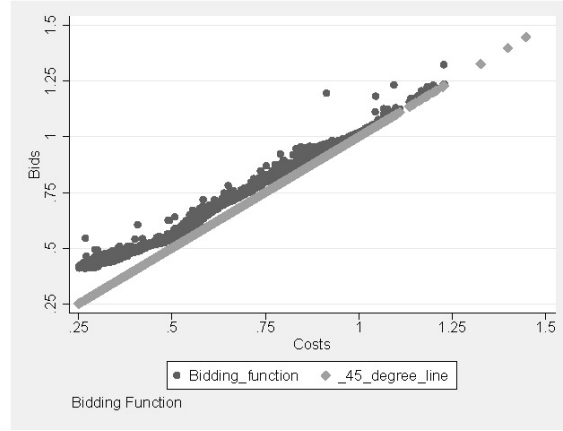


Figure 2.8: Many bidders:  $22 \leq n \leq 28$

on bidders' relative sizes. It is statistically significant (t-value :  $(6.22)^{**}$  in FE regression) that a smaller bidder in an auction is likely to bid with a thinner margin than a larger bidder.

## 2.5 A model for auctions with entry

My stylized entry model considers that a government procures only two projects, *high-end* denoted by  $H$  and *low-end* denoted by  $L$ . There are two groups of firms, large firms denoted by  $BB$ , and SMEs denoted by  $SB$ . Assume every firm has a unit production capacity, regardless of its size. Based on the fact observed in the end of subsection , I suppose the number of large firms is limited to a finite number  $n_{BB}$ , whereas there is an infinitely large number of SMEs. Suppose further that the project  $H$  is so technologically demanding that SMEs are not allowed to bid. The two projects are auctioned through two independent first-price sealed-bid auctions which take place simultaneously.

	OLS	Robust OLS	FE
$x_{ij}$	0.36 (6.49)**	0.36 (6.04)**	0.336 (6.22)**
No. Bidders	-0.082 (86.19)**	-0.082 (49.47)**	-0.081 (80.87)**
Auction date	-0.005 (0.62)	-0.005 (0.65)	
EST <sub>k</sub>	0.002 (3.19)**	0.002 (2.87)**	
logEST <sub>k</sub>	0.008 (0.97)	0.008 (0.91)	
Scoring auction dummy	0.144 (5.72)**	0.144 (3.94)**	
Auction format dummy 1	0.365 (13.53)**	0.365 (9.75)**	
Constant	-2.745 (18.61)**	-2.745 (17.27)**	-2.38 (15.60)**
Observations	7281	7281	7281
R-squared	0.54	0.54	
No. auctions			201

Absolute value of t statistics in parentheses  
 \* significant at 5%; \*\* significant at 1%  
 FE controls auction specific effects.

Table 2.6: Regression Result for bid margins

The procurement proceeds in the following two-stage game: potential bidders decide their entry in the first stage and auctions take place in the second stage. Once a potential bidder enters an auction, it will incur a participation cost  $e$ , obtain its own private information  $c$ , and submit a bid following a Nash bidding strategy in the second stage auction game. I assume  $e$  is identical and common knowledge for all players. Also, no bidder is allowed to participate in both auctions at the same time. If the set-aside program is effective, the low-end project is exclusively offered

to SMEs, so that large firms cannot bid. Otherwise, large firm can be recipients of the  $L$  project.

Despite the simplification, the game has many pure and mixed equilibria depending on the entry process.<sup>27</sup> Therefore, I further assume that entry takes place sequentially as in McAfee and McMillan (1987a), and that the number of bidders is treated as a continuous variable.

Then, the number of players can be reduced into two,  $BB$  and  $SB$ . Each player  $t \in \{BB, SB\}$  decides the number of participants  $n_{s,t}$  in the auction for each project  $s \in \{H, L\}$  subject to the participation constraint, *i.e.*  $n_{L,BB} = 0$  if the set-aside program is effective, and  $n_{H,SB} = 0$ . I also suppose that the player  $BB$  decides her entry first, and successively,  $SB$  makes his participation decision. Once deciding their entry decision  $n_{s,t}$ , each representative player  $t$  incurs participation costs  $e \times n_{s,t}$  for each auction. This setting gives us a unique asymmetric Nash entry equilibrium. The timeline is described in figure 2.9.

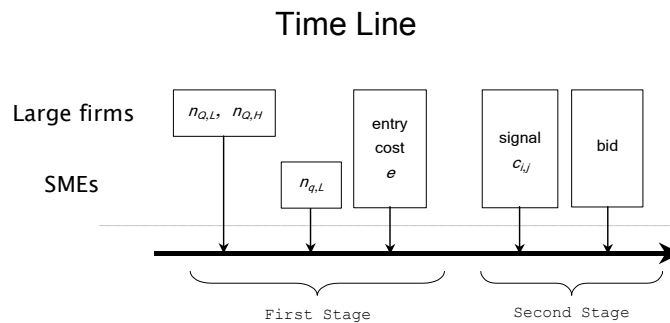


Figure 2.9: The model of auctions with entry

<sup>27</sup>Levin and Smith (1994) show that the number of actual bidders will be stochastic if entry is simultaneous.

## 2.5.1 Analysis for the auction stage

I assume private values, *i.e.*, that one bidder's signal does not affect the other's preferences and that bidders are risk neutral. Each bidder draws her own signal  $\sigma$  which is uniformly distributed on  $[0,1]$ . Let  $c_t(\sigma)$  denote the cost of a type  $t$  bidder, which is increasing and differentiable in  $\sigma$  for each  $t \in \{SB, BB\}$ , with  $c_{SB}(0) = c_{BB}(0) = \underline{c}$  and  $c_{SB}(1) = c_{BB}(1) = \bar{c}$ . Let  $U_t$  be the expected payoff of a bidder in group  $t$  conditional on her signal  $\sigma$ . Also, let  $\Psi_t(b)$  denote the expected payment conditional on his bid value  $b$ . Then, given the number of bidders  $n_{SB}$  and  $n_{BB}$ , the maximization problem of a type  $t$  bidder is given by

$$U_t(\sigma|n_{SB}, n_{BB}) \equiv \max_b p_t(b) - \Psi_t(b|n_{SB}, n_{BB})c_t(\sigma).$$

If  $c_t$  is differentiable and  $\beta_t(\sigma)$  is the bid which maximizes  $p_t(b) - \Psi_t(b|n_{SB}, n_{BB})c_t(\sigma)$ , then one can define  $\psi_t(\sigma) \equiv \Psi_t(\beta_t(\sigma))$ . The envelope integral formula implies that the payoff of a type  $t$  bidder satisfies

$$U_t(\sigma|n_{SB}, n_{BB}) = U_t(0|n_{SB}, n_{BB}) + \int_{\sigma}^1 \frac{d}{d\hat{\sigma}} c_t(\hat{\sigma}) \psi_t(\hat{\sigma}|n_{SB}, n_{BB}) d\hat{\sigma}$$

Then, let  $V_t(n_{SB}, n_{BB})$  be the *ex ante* payoff of a bidder from the auction given  $n_{L, BB}$  and  $n_{L, SB}$ . If I normalize  $U_t(0|\cdot, \cdot)$  as equal to zero, taking expectation for the payoff function  $V_t$  gives

$$V_t(n_{SB}, n_{BB}) = \int_0^1 (1 - \hat{\sigma}) \frac{d}{d\hat{\sigma}} c_t(\hat{\sigma}) \psi_t(\hat{\sigma}|n_{SB}, n_{BB}) d\hat{\sigma}. \quad (2.6)$$

For empirical analysis, I first assume there exists a function  $\tilde{V}_t(\cdot)$  such that

$$\tilde{V}_t(x_t(n_{SB}, n_{BB}), n) \equiv V_t(n_{SB}, n_{BB}),$$

where  $n = n_{SB} + n_{BB}$ . The identity indicates that the *ex ante* expected payoff  $V(\cdot)$  can be decomposed into the two components, i) the number of competitors represented by  $n$ , and ii) the firm size represented by  $x_t$ . The value  $x_t(\cdot)$  in function  $\tilde{V}_t(\cdot)$  is defined in the same manner as in the previous section.<sup>28</sup> By linear approximation,  $\log \tilde{V}_t$  is rewritten as

$$\log \tilde{V}_t(x_t(\cdot), n) = \log \tilde{V}_t(0, 0) + \log \tilde{V}_{1,t}(0, 0) \cdot x_t(\cdot) + \log \tilde{V}_{2,t}(0, 0) \cdot n,$$

where  $\tilde{V}_{1,t} = \frac{\partial \log \tilde{V}_t}{\partial x_t}$  and  $\tilde{V}_{2,t} = \frac{\partial \log \tilde{V}_t}{\partial n}$ . Let  $\log \tilde{V}_t(0, 0) = \alpha_0$ ,  $\log \tilde{V}_{1,t}(0, 0) = \alpha_1$ , and  $\log \tilde{V}_{2,t}(0, 0) = \alpha_2$ . Then, one obtains

$$\log V_t(n_{L,SB}, n_{L,BB}) = \alpha_0 + \alpha_1 \cdot x_t(n_{L,SB}, n_{L,BB}) + \alpha_2 \cdot n. \quad (2.8)$$

The coefficient  $\alpha_1$  represents the bidder  $t$ 's elasticity of *log ex ante* expected payoffs with respect to her relative size  $x_t$ .

<sup>28</sup>Let  $\bar{X}_t$  be the average score of the type  $t$  player, which is given and constant for each  $t \in \{SB, BB\}$ . Also, let  $\bar{X}_L$  denote the bidders' average score in the low-end projects, formulated by  $\bar{X}_L = (\bar{X}_{BB} \cdot n_{L,BB} + \bar{X}_{SB} \cdot n_{L,SB})/n_L$ . According to the definition of  $x$ , the normalized score of type  $t$  firms is given by

$$x_t(n_{L,SB}, n_{L,BB}) = \frac{\bar{X}_t - \bar{X}_L}{\bar{X}_L}. \quad (2.7)$$

The explicit form of (2.7) is given in the appendix.



## 2.5.2 Analysis for an entry equilibrium

Under the set-aside program, large firms may obtain positive rents since their production capacity is limited, whereas the marginal SME bidder obtains zero *ex ante* payoff because of participation by the unlimited number of SMEs. Therefore, a unique entry equilibrium must satisfy

$$\begin{cases} V_{SB}(n_{L,SB}^{r,*}, 0) = e \\ V_{BB}(0, n_{H,BB}^{r,*}) \geq e, \end{cases} \quad (2.9)$$

subject to  $n_{H,BB}^{r,*} \leq n_{BB}$ .

Without set-asides, low-end projects can receive bids from large firms as well. Because of unlimited capacity, the rent of SMEs is still fully extracted. Hence, the SMEs' optimal entry decision  $n_{L,SB}^{u,*}$  satisfies

$$V_{SB}(n_{L,SB}^{u,*}, n_{L,BB}^u) = e,$$

for any  $n_{L,BB}^u$ . Solving (2.10) for  $n_{L,SB}$  gives the SMEs' best response function  $n_{L,SB}^{u,*} = \Gamma(n_{L,BB}^u)$ . Since  $V_{SB}$  is decreasing in both  $n_{L,SB}^u$  and  $n_{L,BB}^u$ ,  $\Gamma'(n_{L,BB}^u) < 0$ . Also, the number of large firms in the market is given and finite and that each bidder with a unit production capacity can bid only once. Therefore, it is legitimate to assume that the number of large bidders in high-end projects  $n_{H,BB}$  is a function of  $n_{L,BB}$ , namely:

$$n_{H,BB} = \Lambda(n_{L,BB}). \quad (2.10)$$

In equilibrium, the *ex ante* payoff of each large bidder must be the same between the two projects. Hence, one obtains

$$V_{BB}(\Gamma(n_{L,BB}^{u*}), n_{L,BB}^{u*}) = V_{BB}(0, \Lambda(n_{L,BB}^{u*})) \quad (2.11)$$

$$\text{subject to } 0 \leq n_{L,BB}^{u*} \leq n_{BB}$$

where the left hand side represents the *ex ante* payoff from low-end projects. This equation gives a unique solution of  $n_{L,BB}^{u*}$ .

### 2.5.3 An empirical model for auctions with entry

According to MLIT (2007), civil engineering projects with their engineer's estimated costs being less than ¥300 million are set aside for SMEs.<sup>29</sup> Consequently, my model considers that a project is high-end if the engineer's estimated cost is no less than ¥300 million, and is low-end if the estimated cost is strictly less than ¥300 million. The proportion of low-end projects in volume account for approximately 61 percent of the total budget for civil engineering contracts during the period.

I then divide the bidders into either large firms or SMEs. In fact, the distinction between SMEs and large firms in the data is a little ambiguous. The set-aside program allows large firms to participate in relatively small projects unless a sufficient competition among SMEs is expected. Consequently, quite a few large firms submit their bids in low-end projects. Also, some SMEs who met a quality standard are

<sup>29</sup>More precisely, ¥300 million is the threshold value with which the government determines whether a project is auctioned for grade B or above contractors, or C or below contractors. Although the contractors with C or below may not satisfy the exact criteria of "SMEs" in Japan, my empirical analysis considers them as SMEs for simplicity.

able to participate in some high-end projects. Hence, a solid way to distinguish these two firm groups might be supposing that the firms who actually bid in the high-end projects are large firms and that the bidders bidding in low-end projects are SMEs. Since the average scores in high- and low-end projects are 1370.9 and 983.3, respectively, I set  $\bar{X}_{SB} = 983.3$  and  $\bar{X}_{BB} = 1370.9$ . Table 2.7 provides the summary statistics of the bidders' scores in both high- and low-end projects.

Project Category	Mean	No. obs	Std Dev.	Max	Min
<i>Low-end</i> < ¥300 mn	983.3	93,808	104.02	1859	475
<i>High-end</i> ≥ ¥300 mn	1370.9	6,777	193.03	1859	848
Total	1017.176	100,585	151.19	1859	475

Table 2.7: Project category

Let  $\bar{n}_{s,t}$  be the number of average bidders with type  $t \in \{SB, BB\}$  in category  $s \in \{H, L\}$  projects. From the data,  $\bar{n}_{L,SB}^r = 7.87$  and  $\bar{n}_{H,BB}^r = 8.20$  are obtained.<sup>30</sup> Denoting by  $\bar{n}_{L,SB}^r$  and  $\bar{n}_{L,SB}^r$  the equilibrium participation under the set-aside program, I conduct a counterfactual simulation to predict  $\bar{n}_{s,t}^u$ .

I first identify the bidders' *ex ante* payoffs. Let  $b_{1,k}$  be the lowest bid in auction  $k$ . Also, with a little abuse of notation, let  $c_{1,k}$  be the estimated cost of the lowest bidder.<sup>31</sup> Since the bid margin is a consistent estimator for the conditional payoff

<sup>30</sup>They are estimated by  $\bar{n}_{s,t} = \frac{1}{I_s} \sum_{\ell=1}^I \mathbf{1}\{s_\ell = s\} \cdot n_{\ell,t}$  for each  $s \in \{H, L\}$ .

<sup>31</sup>Since my model assumes the asymmetric first price procurement auctions, it is possible that the lowest bidder does not have the lowest signal.

$\pi_{1,k}$ , I can define  $\hat{\pi}_{1,k} = b_{1,k} - c_{1,k}$ . Let  $V_{1,k}$  denote the *ex ante* payoff of the lowest bidder in the  $k$ th auction. In the risk neutral environment, it equals the bid margin times the probability of winning, namely

$$V_{1,k} = y_{1,k} \cdot \pi_{1,k},$$

where  $y_{1,k}$  implies the winning probability. The estimation for  $y_{1,k}$  is given by a simple linear probability regression model as follows:

Let  $y$  be the index of awarding where  $y = 1$  if the bidder wins, otherwise  $y = 0$ . The linear probability model is given by

$$y_{i,k} = \delta_1 \frac{1}{n_k} + \delta_2 x_{i,k} + z_k + \nu_k, \quad (2.12)$$

where  $\mathbf{z}_k = (\text{DATE}_k, \text{EST}_k, \log \text{EST}_k, \text{DUMMY}_k)$ .

Table 2.8 shows the regression results of equation (2.12). Fixed effects control the unobserved heterogeneity in project locations. Since the mean difference in scores for SMEs are 39percent lower than that for large firms, I can infer that the mean difference in frequency of winning for SMEs is approximately 5.2 percent lower than that for large firms (t-value : 7.59 with FE).

Furthermore, denoting  $\hat{\delta}$  the least square estimates of (2.12), one obtains the estimated winning probability EST\_PRWIN as

$$\text{EST\_PRWIN}_{1,k} = \hat{\delta}_1 \frac{1}{n_k} + \hat{\delta}_2 x_{1,k}.$$

		OLS	Robust OLS	FE
$x_{ij}$	$(\delta_1)$	0.132 (7.57)**	0.132 (7.59)**	0.132 (7.57)**
(No. Bidders) <sup>-1</sup>	$(\delta_2)$	1.01 (83.35)**	1.01 (87.69)**	1.009 (76.08)**
Auction date		0.001 (0.22)	0.001 (0.23)	0.001 (0.55)
EST <sub>k</sub>		0 (0.16)	0 (0.17)	-0 (0.02)
logEST <sub>k</sub>		0 (0.01)	0 (0.01)	0.001 (0.24)
Scoring auction dummy		0.004 (0.54)	0.004 (0.57)	0.006 (0.72)
Auction format dummy 1		-0.004 (0.56)	-0.004 (0.58)	-0.007 (0.85)
Constant				-0.013 (0.23)
Observations		56704	56704	56704
R-squared		0.23	0.23	0.1

Absolute value of t statistics in parentheses  
\* significant at 5%; \*\* significant at 1%  
Except invited bidders

Table 2.8: Regression Result for linear probability model

Then, a consistent estimator for  $V_{1,k}$  is given by

$$\hat{V}_{1,k} = \text{EST\_PRWIN}_{1,k} \cdot \hat{\pi}_{1,k}.$$

Plugging  $\hat{V}_{1,k}$  into (2.8) and assuming that  $\epsilon_k$  is an *i.i.d.*, mean zero random variable, one obtains

$$\log \hat{V}_{1,k} = \alpha_0 + \alpha_1 x_{1,k} + \alpha_2 n_j + \alpha_3 z_k \epsilon_j.$$

Denoting by  $\hat{\alpha}_1, \dots, \hat{\alpha}_3$  the least square estimates of  $\alpha$  and taking expectation on both sides, one obtains

$$E(\log \hat{V}_1) = \hat{\alpha}_1 + \hat{\alpha}_2 E(x_1) + \hat{\alpha}_3 E(n).$$

The regression results are shown in Table 2.9. To obtain the model for simulation, I replace the expectations by  $\bar{V} = \frac{1}{m} \sum_k \log V_{1,k}$ ,  $\bar{x} = \frac{1}{m} \sum_k x_{1,k}$  and  $\bar{n} = \frac{1}{m} \sum_k n_k$ . Furthermore, I assume that this equation holds for each group of bidders and each type of project. Let  $\bar{V}_{s,t}$  and  $\bar{x}_{s,t}$  represent the average *ex ante* log payoff and the score of type  $t$  winning bidders in category  $s$  projects. Then, I suppose

$$\bar{V}_{s,t} = \hat{\alpha}_1 + \hat{\alpha}_2 \bar{x}_{s,t} + \hat{\alpha}_3 \bar{n}_s \tag{2.13}$$

holds for any  $s$  and  $t$ , where  $\bar{n}_s = \bar{n}_{s,SB} + \bar{n}_{s,BB}$ .

Equation (2.13) constitutes the counterfactual simulation, where  $\hat{\alpha}_2$  captures the marginal effect of bidder's size on the profitability, and identification for the

	OLS	Robust OLS
$x_{ij} (\alpha_1)$	1.361 (17.20)**	1.361 (16.81)**
No. Bidders ( $\alpha_2$ )	-0.204 (149.03)**	-0.204 (64.35)**
Constant ( $\alpha_0$ )	-3.662 (17.39)**	-3.662 (15.75)**
Observations	7281	7281
R-squared	0.77	0.77
Absolute value of t statistics in parentheses		
* significant at 5%; ** significant at 1%		
Except invited bidders		

Table 2.9: Regression Result for expected payoffs

participation cost  $e$ . Plugging it into (2.9) gives

$$\log \hat{e} = \hat{\alpha}_1 + \hat{\alpha}_3 \bar{n}_{SB,L}^r.$$

The individual rationality condition for large firms must be checked with (2.13) and (2.9). Let  $\bar{n}_{H,BB}^r$  be the number of large firms in high-end projects. Then,

$$\hat{\alpha}_1 + \hat{\alpha}_3 \bar{n}_{H,BB}^r \geq \log e.$$

## 2.6 Simulation

### 2.6.1 The model

Consider first the follower's problem in the entry game. Plug (2.7) into (2.13), and one obtains the *ex ante* payoff function  $V(\cdot)$  of SMEs such that

$$\bar{V}_{SB}(\bar{n}_{L,SB}^u, \bar{n}_{L,BB}^u) = \hat{\alpha}_1 + \hat{\alpha}_2 \cdot \bar{x}_{SB}(\bar{n}_{L,SB}^u, \bar{n}_{L,BB}^u) + \hat{\alpha}_3 \cdot (\bar{n}_{L,SB}^u + \bar{n}_{L,BB}^u). \quad (2.14)$$

$\bar{V}_{SB}(\bar{n}_{L,SB}^u, \bar{n}_{L,BB}^u) = e$  holds in equilibrium. Solving this for  $n_{SB}$  gives the explicit form of the best response function  $\Gamma(\cdot)$ . The complete derivation is provided in the appendix.

Next, the *ex ante* payoff of large firms in the low-end project is given by

$$\bar{V}_{BB}(\bar{n}_{L,SB}^u) = \hat{\alpha}_1 + \hat{\alpha}_2 \cdot \bar{x}_{BB}(\Gamma(\bar{n}_{L,SB}^u), \bar{n}_{L,SB}^u) + \hat{\alpha}_3 \cdot (\Gamma(\bar{n}_{L,SB}^u) + \bar{n}_{L,SB}^u),$$

which is expressed as a function of  $\bar{n}_{L,SB}^u$ . Therefore, equation (2.12) in the simulation study becomes

$$\hat{\alpha}_2 \cdot \bar{x}_{BB}(\Gamma(\bar{n}_{L,SB}^u), \bar{n}_{L,SB}^u) + \hat{\alpha}_3 \cdot (\Gamma(\bar{n}_{L,SB}^u) + \bar{n}_{L,SB}^u) = \lambda \cdot \hat{\alpha}_3 \cdot \Lambda(\bar{n}_{L,SB}^u). \quad (2.15)$$

The left hand side describes the *ex ante* expected payoff from low-end projects, whereas the right hand side represents that from high-end projects. Since low-end projects are greater (in value terms) than high-end projects, I introduce a weight variable  $\lambda$  so that (2.15) describes an equilibrium in which the large firm's gain from entering the low-end market is identical to that from entering the high-end market.



For simplicity in simulation calculation, I linearize  $\Gamma(\cdot)$  and  $\Lambda(\cdot)$  in equation (2.15). The details are described in the appendix.

Finally, I describe the comparative statics of the winning bid with respect to the participation restriction. Let  $b_{1,k}$  be the lowest bid in auction  $k$ . For any  $k = 1, \dots, m$ , the distribution of  $b_{1,k}$  is written as<sup>32</sup>

$$G_{b_{1,k}}(b_{1,k}|x_k, n_k, z_k) = \prod_{\tau=1}^{n_k} \{1 - G_{b_{\tau,k}}(b_{1,k}|x_{\tau,k}, n_k, z_k)\}.$$

That is,  $b_{1,k}$  is a random variable, given the numbers of bidders, the normalized score of each bidder and exogenous variables such as the auction specific effect.

To know the effect of the winning bidder's size on the winning bid, I set up a linear regression model for the lowest bids. Assuming that  $\epsilon_{b_1}$  follows an *i.i.d.* distribution, the model is given as

$$b_{1,k} = \beta_0 + \beta \cdot x_{1,k} + \beta_2 \cdot n_k + \beta_3 \cdot \mathbf{z}_k + \epsilon_{b_1}, \quad (2.16)$$

where  $n_k$  and  $z$  control for the number of bidders and other auction specific effects, respectively. Then,  $\hat{\beta}_1$  measures the difference of the winning bid between large firms and SMEs.

Table 2.10 shows the result of the regression of the lowest bids on  $x$ . The bidder with the lowest bid awards the project for each conditional on the bid being equal to or lower than the reservation price. Again I denote by  $b_{1,k}$  the lowest bid in auction  $k$ . Fixed effects control the area specific effects. This regression indicates that the

<sup>32</sup>Recall that the bidding function depends upon  $b_{i,k}$ ,  $x_{i,k}$ ,  $n_k$  and  $z_k$ , and  $c_{i,k}$  is a random variable subject to  $F_{c_i}(c_{i,k}|\mathbf{z}_k)$ .

	OLS	Robust OLS
$x_{ij} (\alpha_1)$	-0.083 (4.88)**	-0.07 (4.79)**
No. Bidders ( $\alpha_2$ )	-0.003 (11.43)**	-0.004 (14.85)**
Auction date	-0.121 (51.55)**	-0.122 (51.55)**
EST <sub>k</sub>	0.001 (3.18)**	0 (3.18)**
logEST <sub>k</sub>	-0.068 (29.72)**	-0.062 (29.72)**
Scoring auction dummy	-0.065 (8.44)**	-0.054 (8.44)**
Auction format dummy 1	-0.014 (1.63)	-0.007 (1.63)
Constant	2.264 (52.75)**	2.157 (52.75)**
Observations	7728	7728
R-squared	0.43	0.48

Absolute value of t statistics in parentheses  
\* significant at 5%; \*\* significant at 1%  
Except invited bidders

Table 2.10: Regression Result for lowest bids

winning bid decreases as the number of bidders increases or the bidder's score  $x$  is higher.

Finally I derive the mean winning bids in the low-end projects, using the above regression. Under the set-aside program, only SMEs are the bidders in the low-end projects. Hence, denoting by  $\bar{b}_{1,L}^r$  the mean lowest bids,

$$\bar{b}_{1,L}^r = \beta_0 + \beta_2 \cdot n_{L,SB}^r.$$

In case of unrestricted participation, both large firms and SMEs will win the low-end project with probabilities equal to  $\hat{y}_{1,BB} \cdot \bar{n}_{L,BB}$  and  $\hat{y}_{1,SB} \cdot \bar{n}_{L,SB}$ . Define  $\bar{n}_L^u \equiv \bar{n}_{L,BB} + \bar{n}_{L,SB}$  as the mean number of bidders in low-end projects. Then, I assume that the mean winning bids of large firms and SMEs can be described by

$$\begin{aligned} \bar{b}_{1,BB}^u &= \hat{\beta}_0 + \hat{\beta}_1 \cdot \bar{x}_{BB}(\cdot) + \hat{\beta}_2 \cdot \bar{n}_L^u, \\ \text{and } \bar{b}_{1,SB}^u &= \hat{\beta}_0 + \hat{\beta}_1 \cdot \bar{x}_{SB}(\cdot) + \hat{\beta}_2 \cdot \bar{n}_L^u \end{aligned}$$

Then, the mean winning bids in the low-end projects is thus given as

$$\bar{b}_{1,L}^u = \bar{b}_{1,BB}^u \cdot \hat{y}_{1,BB} \cdot \bar{n}_{L,BB} + \bar{b}_{1,SB}^u \cdot \hat{y}_{1,SB} \cdot \bar{n}_{L,SB}.$$

The marginal effect by restricting participation is thus given by  $\bar{b}_{1,L}^r - \bar{b}_{1,L}^u$ .

## 2.6.2 Results

The empirical results suggest that the set-aside program likely decreases procurement costs. A counterfactual simulation predicts what the bidder's entry decision

and bidding behavior would be were the program to be eliminated. The program yields the competing effects in terms of government procurement costs, the cost reduction in set-aside projects and the cost increase in the remaining projects.

The simulation study suggests that, were the program to be eliminated, 3.54 large firms on average would switch their entry from high-end to low-end projects so that their *ex ante* payoff from these two projects must be identical in equilibrium. Since there is the difference in volume for each category of projects, represented by  $\lambda = 0.65$ , mean number of large firms in low-end projects would be 2.28, which is obtained by 3.54 times  $\lambda$ .

The serious problem by removing the participation restriction is that the participants would decrease in both high- and low-end projects. In high-end projects, the number of large firms would drop from 8.20 to 4.66, which would raise the procurement costs of those projects by 1.4 percent. At the same time, the large firms' participation in the low-end projects would depress SMEs' entry into the low-end projects. The mean number of SME participants would decline from 7.87 to 4.86.<sup>33</sup> The number of both large firm and SME participants in low-end projects would drop from 7.87 to 7.14 on average since, according to the static entry model, the participation of one more large firm in the low-end projects would reduce 1.32 SME participants on average.<sup>34</sup> The procurement costs of low-end projects would fall by 47 percent, despite the fewer participants, because of the entry by cost efficient large firms. The average score of bidders would be increased from 983.3 to 1107.2.

<sup>33</sup>This outcome implicitly assumes that each group of bidders follows a Nash equilibrium bidding strategy. Should the large firms intentionally make a low-ball bid to deter entry by SMEs, the decrease of SMEs would be much more significant.

<sup>34</sup>The coefficient is given by  $\gamma = 1.32$ .

Surprisingly, the resulting lack of competition would drive up government procurement costs. There are two competing effects set-asides create on government procurement costs; increasing competition versus the participation of cost inefficient SMEs. Taking also into the fact that the government spent approximately 60 percent of the procurement budget to low-end projects, the effect of increasing competition would overcompensate the effect of production inefficiency cost. The simulation study suggests that set-asides would decrease government procurement costs by 0.28 percent.

Project category	Set-asides		Unrestricted	
	Low	High*	Low	High*
Mean no. small bidders	7.87	0	4.86	0
Mean no. large bidders	0	8.20	2.28	4.66
Mean no. total bidders	7.87	8.20	7.14	4.66
Mean Scores	983.3	1370.9	1107.2	1370.9
Procurement Cost Change	-	-	-0.44%	1.41%
Overall effect	-	-	0.28%	
Project Volume (¥ bn.)	1319.16	851.80	1319.16	851.80
Sum of engineer's estimates (Share %)	(61.0)	(39.0)	(61.0)	(39.0)
Entry Costs (% of engineer's estimates)	0.95%		0.95%	
Profits (large firms) (% of engineer's estimates)	-	0.96%	1.79%	1.79%

\*High-end Projects are the project whose engineer estimated cost is no less than ¥300 million.

Table 2.11: Estimation for the effect of set-asides

It may be interesting to observe how the *ex ante* expected profits of large firms are changed by set-asides. Without set-asides, large firms obtain a positive expected gain (1.82 percent of the engineer's estimated cost for each auction), and the net positive gain from entry is almost 1 percent of the project estimated cost. Set-asides completely squeeze the positive net gain from the large firms, so that the expected gain of large firms with set-asides is almost zero (0.01 percent). Obviously, this rent extraction from large firms contributes to lowering government procurement costs, more than offsetting the resulting production cost inefficiency.

## 2.7 Conclusion

Set-asides are widely used in real world public procurement. The encouragement of SMEs has evoked a controversy on how much society is paying extra costs. However, there is no previous systematic analysis to measure the impact on procurement costs.

This paper provides the first systematic analysis of the effect of small business set-asides on government procurement costs, bidding behaviors, and bidder participation in competitive bidding processes. The simulation study suggests that the program dramatically increases the SMEs' participation but is almost neutral with respect to the procurement costs. The production inefficiency caused by set-asides is overcompensated by the increased entry and resulting enhancement of competition by large firms. I found that the set-aside program would increase SMEs' participation in the procurement auctions by approximately 40 percent.

The empirical results show that the set-aside program has been successful. It improves equity between advantaged and disadvantaged firms and reduces government

procurement costs. The results also suggest that the government cost of set-aside auctions is exaggerated if one considers only the excess amount on contracts allocated to SMEs. The theoretical literature has suggested that despite the efficiency loss, the encouragement of less advantaged bidders in the auction can reduce procurement costs. For instance, Bulow and Roberts (1989) and McAfee and McMillan (1989) insist that bidding credits (or bid discounts in procurement auctions) for disadvantaged bidders increase the auctioneer's welfare, yielding more competitive pressure on advantaged bidders. Similarly, the subsidized SMEs drive non-subsidized bidders to give up more of the gain on the contracts they award.

The conclusion also provides an economic rationale on why several countries such as the United States and Japan opt out of SMEs from the Government Procurement Agreement (GPA) of the World Trade Organization (WTO). Although Article 4 in GPA prohibits the member countries to give unfavored treatment for any company, the set-aside programs are exempted in the GPA Appendix. EU countries have also been renegotiating with the WTO to obtain the exclusion of their SMEs. An important question, however, is whether those practices are robust to corruption or favoritism. Further theoretical and empirical consideration are needed.

A restriction of this paper is that it does not consider the long term effect of set-asides. In the long run, there are positive and negative effects of set-asides on procurement costs. If SMEs could win more auctions, they would have more chances to develop their production skills through learning by doing. On the other hand, subsidization of SMEs may discourage them to develop their businesses to the stage where they cannot be favored in the preference program. Given the sheer volume

of public sector procurement, it is clear that more serious research and evaluation is needed to investigate the long run effect of the set-aside program.



## CHAPTER 3

# PROCUREMENT AUCTIONS WITH PRE-AWARD SUBCONTRACTING

### Abstract

To be the lowest bidder in procurement auctions, prime contractors commonly solicit subcontract bids at the bid preparation stage. A remarkable feature of the subcontract competition is that “winning is not everything”; the lowest subcontractor gets a job conditional on his prime contractor’s successful bid. This paper makes the first attempt to establish a model for such pre-award subcontract competitions included in procurement auctions. I find that subcontractors strategically provide larger discounts on their bids in response to increasing competition among prime contractors. The contribution of this paper is to clarify that the behavior results in an endogenous downward shift in the distribution of bidders’ costs as the number of rivals increases, or the reservation price drops, unlike the case in the standard mechanism design model where the distribution of bidder’s private information is independent from such competitive environment variables. As a result of the theory, I demonstrate that the revenue maximizing reservation price is decreasing in the number of bidders. It is also shown that, if the prime contractors’ endogenous

participation in the auction is taken into account, subsidizing the potential bidders' entry is a remedy to solve the double marginalization problem, allowing the auctioneer to extract more rents from subcontractors.

### 3.1 Introduction

Subcontracting and out-sourcing are common business practices in procurement markets. In a highway construction project, for instance, the winning bidder may subcontract road marking or signal work to specialty firms. In addition, the contractor may purchase raw materials or equipment from other sources, which can also be considered as subcontracting in the broader sense. In the construction projects which cover a wide range of work, it is impractical for the contractor to perform the entire work by itself. Therefore, for prime contractors the bulk of the project costs consists of subcontract payments in large construction projects.

To obtain qualified subcontracts for fair prices, prime contractors commonly solicit irrevocable pre-bid price quotes from subcontractors in the bid preparation stage. This practice also satisfies the prime contractor's needs to estimate the project cost for tendering<sup>35</sup> or the requirement by some procurement buyers for contractors to submit a proposed subcontracting plan which must be approved by the contracting officer before the contract is awarded.<sup>36</sup> Such pre-award subcontract

<sup>35</sup>There are several legal issues regarding the irrevocability of subcontract bids. California Supreme Court case of *Drennan v. Star Paving Co.*,<sup>2</sup> suggested that a subcontract bid is irrevocable once it is relied on by the prime contractor in computing her overall bid (the Drennan rule). However, the prime contractor are free to lower the subcontract price after being awarded with disclosing the current lowest subbids on some subcontracts to obtain the further lower price (bid shopping). See Grosskopf and Medina (2007) for more detail.

<sup>36</sup>For instance, the state of Oregon enforces bidders in public projects to submit a list of First-Tier subcontractors and their bids if the amount of the bid is greater than five percent of total project bid or 15,000 (ORS 279C.370).

competitions are commonly seen not only in public procurement, but also in private markets (Dyer and Kagel (1996), Degn and Miller (2003)).

The aim of this research is to model subcontract auctions included in the procurement auction. The key assumption is that the prime contractor's cost consists of not only her marginal cost to perform the work herself, but also the payments to the subcontractors selected through the upstream auctions. A remarkable feature of the subcontract auctions is that winning is not everything; the awarded subcontractor gets a job only if his prime contractor wins in the downstream competition. Presumably, analyzing subcontract auctions requires a slightly different framework than standard auctions where the lowest bidder always obtains a payoff.

There are several open questions attributable to modeling upstream competitions that occur prior to the downstream auction. Firstly, one might wonder how to characterize the subcontractor's objective function. Since a subcontractor obtains a payoff only if his prime contractor wins in the downstream auction, subcontractors would also be concerned about the competition in the downstream auction. In particular, if the prime contractor faces an intense competition in the downstream auction, lowering the subcontract bid will help the prime contractor win in the downstream competition, and could, in turn, be beneficial for the subcontractor.

Secondly, the existing auction literature provides an unclear guidance about the bidder's information acquisition process and bidding behavior in the downstream auction if there exists pre-award subcontract auctions. More specifically, suppose the number of bidders or reservation price in the downstream auction affects the subcontractors' behavior in upstream auctions. Then, it could follow that these effects have an influence back in the downstream auction due to the fact that the

bidder's cost consists of the winning bids in upstream auctions. In particular, prime contractors may have a stronger bargaining power as competition increases in the downstream auction. Thus, one need to verify whether the existing auction theorem (such as revenue equivalence, optimal reservation price) still holds in the downstream auction.

Thirdly, prime contractors choose a subcontractor using any methodology. So, the resulting question is what is the optimal mechanism for a prime contractor to select subcontractors, especially regarding the selection between first- and second price rule. Taking into account the widespread use of first-price auction in the real world subcontract competition, it might be interesting to examine whether the first-price auction dominates the second-price rule from the viewpoint of the prime contractor's profit maximization.

The final and most basic question lies in whether there exists a symmetric increasing equilibrium which supports the above arguments. Since the bidder's objective function would differ from the standard auction setting, especially in upstream auctions, one need a new model to analyze the contractors' behaviors in the procurement auction including pre-award subcontract competitions.

To answer these puzzles, I establish the following two-stage game: in the first stage each prime contractor solicits irrevocable price offers from her own subcontractors, and selects the lowest one, assuming that prime contractors know the distribution of the subcontractors' marginal costs, but not the values. With the selected subcontractor, each prime contractor makes a pre-award subcontract agreement which specifies the job scope performed by the subcontractor and the payment made by the prime contractor to the subcontractor. At the second stage, the prime

contractor computes the total project cost by adding up her own marginal cost and the subcontract payment. Then, each prime contractor bids in the procurement auction assuming that each bidder does not know the other bidders' project costs. The winning prime contractor undertakes the contract with the selected subcontractor, incurring her own marginal cost and making the subcontract payment.

Verifying the existence of a symmetric increasing equilibrium in both upstream and downstream auctions, I demonstrate that prime contractors have a larger bargaining power than subcontractors in response to increasing competition in the downstream auction; subcontractors strategically provide larger discounts on their price offers as the number of prime contractors rises, or the reservation price declines in the downstream competition. The behavior creates an endogenous downward shift in the distribution of the bidders' private information as the number of bidders increases, or the reservation price drops in the downstream auction. This contradicts the assumption employed in standard auction models that the distribution of the players is exogenously given.

Furthermore, I show that prime contractors' expected payoffs may be greater when they use the first-price auction than the second-price auction in upstream competitions. It is also demonstrated that the use of the first-price auction in upstream auctions benefits not only the procurement buyer, but also society, yielding more *ex post* efficient allocation. These results are fairly in line with the conventional wisdom on the widespread use of the first-price auction in real world procurement auctions.

Given the bidders with endogenously distributing private information, what about the optimal mechanism for the procurement buyer? It is easily shown that

the Revenue Equivalence Theorem still holds in the downstream auction due to the fact that the independent private value (IPV) property is well-maintained. However, the optimal reservation price becomes a function of the number of bidders, contrary to the optimal reserve price theorem in the standard IPV environment introduced by Riley and Samuelson (1981).

The most striking result will be drawn when our theory is applied to the model of auctions with endogenous participation. It is well-known that for entry to be optimal (revenue maximizing) and hence efficient,<sup>37</sup> the private gain from further entry must equal the social gain (See *e.g.* Levin and Smith (1994)). With pre-award subcontracting, however, the prime contractor cannot capture the positive effect caused by the additional discount of subcontract. The prime contractor's gain from entry is strictly less than the social gain. Hence, entry is likely to end up being insufficient if the pre-award subcontract competition is taken into account.

As for related articles, our model is the closest to Hansen (1988), where he argues that bidders bid more aggressively if there is a downstream market in which the quantity demanded is determined by the winning bid price. A non-trivial extension I make to Hansen lies in modeling the downstream competition with using an auction game. It enables us to provide qualitative examination on the downstream market, such as optimal design and efficiency analysis.

On the other hand, there is a fairly large volume of existing literature which deals with procurement auctions with subcontracting. For instance, Kamien et al. (1989) and Gale et al. (2000) analyze post-award subcontracting in which the winning firm

<sup>37</sup>If potential bidders are *ex ante* identical, and if non-positive reservation price is set, then in a mixed strategy equilibrium each potential bidder is indifferent between entry or staying out. Therefore, the total surplus is exactly the same as the auctioneer's welfare.

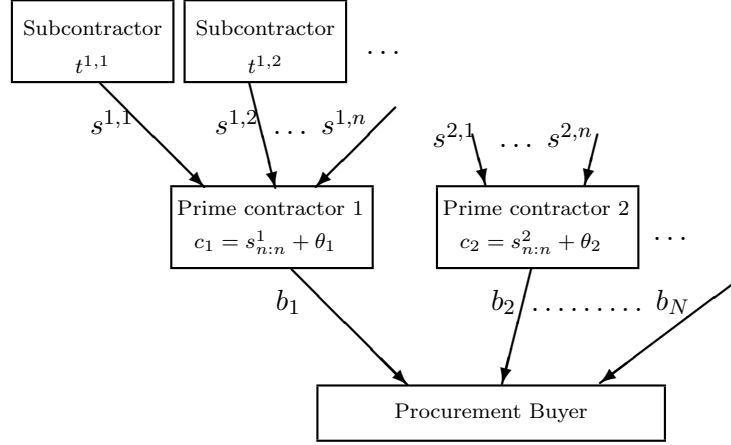
may split the award and subcontract parts of it to the best partners. Because the subcontractors' marginal costs are essentially unobservable for prime contractors, there exists an adverse selection problem between sub- and prime contractors. A great deal of literature, therefore, investigates the principal-agency relationship between sub- and prime contractors (*e.g.* Kawasaki and McMillan (1987)).<sup>38</sup> Furthermore, the variation of subcontracting regulation across nations is precisely discussed in Marechal and Morand (2003). However, the existing literature neither models both upstream and downstream competitions, nor analyzes both at the same time.

The remaining part of this paper is organized as follows. In section 3.2, I describe the model of procurement auctions with pre-award subcontracting. Section 3.3 examines the equilibrium bidding behavior in upstream auctions. Section 3.4 is devoted to the equilibrium analysis in the downstream auction. Section 3.5 provides further discussion, and the final section concludes. Proofs are given in an appendix.

### 3.2 The model

Consider a government auctions a project to  $N$  prime contractors denoted by  $i = \{1, \dots, N\}$ . Suppose the value of the project for the government is equal to  $V > 0$ . We assume that an winning prime contractor can complete the project only by subcontracting a certain portion of the project. For simplicity, I suppose that the proportion of subcontracting is publicly known, fixed, and identical for any prime contractor.

<sup>38</sup>Also a variety of issues on procurement contracts are discussed in Bajari and Tadelis (2001), such as contract forms (fixed-price or cost-plus), tendering or negotiation, change orders, default or non-performance.



Suppose there are  $n \times N$  subcontractors. The  $i$ th prime contractor has  $n$  subcontractors indexed by  $j$ . We further assume that the set of subcontractors transacted with  $i$ th prime contractor is disjoint with the set of subcontractors transacted with  $i'$ . Once the government announces the auction, each prime contractor specifies the job scope of the subcontract, sets the assignment rule  $\psi_j^i : [0, 1]^n \rightarrow [0, 1]$  and the payment rule  $\xi_j^i : [\underline{t}, \bar{t}]^n \rightarrow [0, \infty)$  for all  $j$ , and solicits pre-bid price quote  $s$  from their subcontractors. Based on the assignment rule and price quotes, the prime contractor selects a subcontractor, and makes the contingent subcontract on which both parties precommit to the job scope and the payment amount. Finally, after calculating her marginal costs, the prime contractor submits a bid, denoted by  $b \in \mathfrak{R}$ , in the procurement auction. The government selects a prime contractor following the assignment rule  $X_i : [0, \infty)^N \rightarrow [0, 1]$  for all  $i$ . So,  $\sum X_{i=1}^N < 1$  if the reservation price  $b_r$  is strictly less than the project value  $V$ . If the  $i$ th prime contractor wins the project, he subcontracts the given portion of the project to the subcontractor and makes a payment  $\xi_i$ .



Throughout the paper, I assume private information. The marginal cost for each subcontractor to perform the subcontracted work is characterized by  $t \in [\underline{t}, \bar{t}]$ , which is privately known to the subcontractor, and drawn from an identical and publicly known distribution function  $F_t$ . The total cost  $c_i$  of the  $i$ th prime contractor is represented by the sum of the marginal cost for the part of the work performed by herself and the payment for subcontracting. That is, it satisfies  $c_i = \theta_i + \xi_i$ , where  $\theta_i$  is the prime contractor's marginal cost. We assume that  $\theta_i$  is drawn from a commonly known cumulative distribution  $F_\theta$ , but the exact amount of  $\theta_i$  is known only to the  $i$ th prime contractor and not to any other players including her own subcontractors. Also,  $\xi_i$  is privately known only to the prime contractor  $i$  and the awarded subcontractor, so that the  $i$ th prime contractor's cost  $c_i$  is also private information in the procurement auction game.

To simplify the analysis for the existence of an increasing symmetric equilibrium, I assume that the cumulative distribution functions of  $\theta$  and  $t$  satisfy increasing hazard rate (IHR) *i.e.*  $\frac{d}{d\theta} \frac{f_\theta(\theta)}{F_\theta(\theta)} \leq 0$  and  $\frac{d}{dt} \frac{f_t(t)}{F_t(t)} \leq 0$ . Many functions including uniform, normal, and chi distributions meet the criteria.

### 3.3 Equilibrium in upstream auctions

From now on, I analyze a symmetric equilibrium in upstream auctions with two cases. The first subsection discusses a simple case where prime contractors' costs only depend on the selected subcontract bids, and the number of subbids solicited is equal to one. The second subsection is devoted to the analysis with considerable generality where prime contractors' costs consist of their own marginal

costs plus payment to subcontracts. Both analyses will proceed backward, assuming the symmetric equilibrium in the downstream (second-stage) auction.

### 3.3.1 A simple case

In a simple model, I assume that  $n = 1$ , and that the  $i$ th prime contractor's marginal cost  $\theta_i$  is constant and is normalized to be zero. We first examine the case where the first-price rule is used to allocate the pre-award subcontract.<sup>39</sup>

Let  $t_i$  denote the cost signal drawn by the subcontractor of the  $i$ th prime contractor. Because of the first-price rule in subcontract auction, the lowest subbid is equal to the amount of payment for the subcontractor. Hence, assuming that all the subcontractors follow a symmetric increasing strategy  $\sigma$  in subcontract competition, the prime contractor's total marginal cost can simply be described by  $c = \sigma(t_i)$ . Based on the characterization on private information, each prime contractor strategically determines her optimal bid price. Given the symmetric increasing strategy  $\sigma$ , the cost of the  $i$ th prime contractor is independently and identically distributed for all  $i$ . Hence, standard arguments can be applied to claim that there exists a symmetric increasing equilibrium  $\beta$  in the prime contractor's strategy if a standard auction mechanism is used in the procurement auction.<sup>40</sup>

Now, we go back to the first stage competition in subcontracting. Suppose all the other subcontractors including those of rival prime contractors follow the symmetric increasing strategy  $\sigma(\cdot)$  upon offering their pre-bid price quote. Then, the subcontractor's winning probability conditional on his subbid equal to  $s$  is given

<sup>39</sup>We will relax the constraint in the generalized model.

<sup>40</sup>Standard auctions include the first- and second-price auctions in which the prime contractor with the lowest marginal cost is awarded.

by

$$\underbrace{\Pr(\text{Wins subcontract}|s)}_{P(s)} = 1.$$

Furthermore, conditional on his winning subcontracting, his prime contractor beats the remaining  $N - 1$  rival contractors with probability

$$\underbrace{\Pr(\text{Prime wins}|P(s), s)}_{Q(s|N, b_r)} = [1 - F_t(\sigma^{-1}(s))]^{N-1},$$

assuming that every prime contractor follows the symmetric increasing bidding strategy  $\beta$ . If the  $i$ th prime contractor is awarded, the  $i$ 's subcontractor finally receives the payment  $s$  from his prime contractor and incurs the marginal cost  $t_j^i$  to complete the subcontract work. Therefore, we obtain the subcontractor's objective function as

$$\begin{aligned} \pi(t_j^i|N, n, b_r) &= \max_s (s - t_j^i) P(s) Q(s|N, b_r) \\ &= \max_s (s - t_j^i) [1 - F_t(\sigma^{-1}(s))]^{N-1}. \end{aligned} \quad (3.1)$$

To solve the symmetric equilibrium  $\sigma$ , we take derivative with respect to  $s$ . After solving the differential equation and suppressing some notations, one obtains the candidate of the subcontractor's symmetric equilibrium strategy  $\sigma$  as follows

$$\sigma(t|N, b_r) = t + \frac{\int_t^{b_r} [1 - F_t(\hat{t})]^{N-1} d\hat{t}}{[1 - F_t(t)]^{N-1}}.$$

This is identical to the equilibrium bidding strategy in a symmetric independent private value procurement auction with four bidders, implying that each subcontractor bids as if the subcontractors under the other prime contractor are also his rivals.

It is easy to see that subcontractors bid more aggressively as increasing competition among prime contractors; If one obtains the symmetric equilibrium strategy  $\sigma$  in the case with  $N - 1$ , the second term of the strategy will be replaced as  $\frac{\int_t^{b_r} [1-F_t(\hat{t})]^{N-2} d\hat{t}}{[1-F_t(t)]^{N-2}}$ . So, subcontractors' informational rents decline anywhere on  $t \in [t, \bar{t}]$  as  $N$  changes from  $N - 1$  to  $N$ ,<sup>41</sup> or reservation price  $b_r$  is lowered.

When one generalizes this simple case, the cost distribution of prime contractors is still endogenously determined by the number of rivals and the reservation price. The qualitative features of the simple example are therefore robust.

### 3.3.2 Generalized cases

Now I investigate the generalized case where  $\theta$  is also a random variable.<sup>42</sup> Suppose the prime contractor with the lowest cost wins the project. Also, I assume that the prime contractor never use a reservation value upon soliciting price quotes. Then, the subcontractor's objective function is given as

$$\pi^i(t_j^i | N, b_r) = \max_s (s - t_j^i) \underbrace{\Pr\{Wins subcontract\}}_{P(s|n)} \underbrace{\Pr\{Prime wins | P(s|\cdot)\}}_{Q(s|N, b_r)}. \quad (3.2)$$

<sup>41</sup>For any  $\hat{t} > t$ ,  $\left(\frac{1-F_t(\hat{t})}{1-F_t(t)}\right)^{N-1} - \left(\frac{1-F_t(\hat{t})}{1-F_t(t)}\right)^{N-2} = \left(\frac{1-F_t(\hat{t})}{1-F_t(t)}\right)^{N-2} \left[\frac{1-F_t(\hat{t})}{1-F_t(t)} - 1\right] < 0$ . Thus, for any  $t < b_r$ , the bidder's informational rent decreases as the number of prime contractors increases from  $N - 1$  to  $N$ .

<sup>42</sup>The randomized  $\theta$  can capture not only the private information on the prime contractors. But it also takes into consideration the case where prime contractors may subcontract other parts of works and these subcontracts are competitively distributed to other sets of subcontractors.

Assuming that all the other subcontractors follow the increasing strategy  $\sigma$ ,  $P$  is given as

$$P(s|n) = [1 - F_t(\sigma^{-1}(s))]^{n-1} .$$

On the other hand,  $Q(s|\cdot)$  is the conditional probability with which his prime contractor wins the auction given that his bid  $s$  is the lowest and the prime contractor's marginal cost equals  $\theta_i$ . This happens if  $s$  is lower than the sum of the three random variables,  $\theta_i$ ,  $\theta_{i'}$  and  $\sigma(t_{1:n})$  each of which are independently distributed.

To obtain the distribution, we first explore the probability with which the prime contractor  $i$ 's cost  $c_i$  is lower than the another prime contractor's cost  $c_{i'}$  given that the  $i$ 's own marginal cost  $\theta_i$  and the payment to the subcontract is equal to  $s$ . Denote  $F_c(\cdot)$  the cumulative distribution function of  $c$  in the symmetric equilibrium. Then, by the convolution theorem, the probability is given by<sup>43</sup>

$$1 - F_c(s + \theta_i|\sigma) = \int_{\underline{t}}^{\bar{t}} n f_t(t_{1:n}) [1 - F_t(t_{1:n})]^{n-1} [1 - F_\theta(s + \theta_i - \sigma(t_{1:n}))] dt_{1:n}, \quad (3.3)$$

and the density of  $F_c$  is given by

$$f_c(s + \theta_i|\sigma) = \int_{\underline{t}}^{\bar{t}} \{n f_t(t_{1:n}) [1 - F_t(t_{1:n})]^{n-1}\} f_\theta(s + \theta_i - \sigma(t_{1:n})) dt_{1:n}.$$

There are  $N$  other prime contractors, each of which draws  $\theta_i \in \{1, \dots, N\}$  from the common distribution  $F_\theta$ . Therefore, the probability  $Q$  with which the  $i$ th prime contractor wins the contract provided that the selected subcontract price is equal

<sup>43</sup>An alternative way to obtain  $1 - F_c(\cdot)$  is given in appendix.

to  $s$  is given by

$$Q(s|N, b_r, \sigma) = \int_{\underline{\theta}}^{b_r - s} [1 - F_c(s + \theta_i)]^{N-1} f_{\theta}(\theta_i) d\theta_i, \quad (3.4)$$

and the derivative is

$$Q'(s|N, b_r, \sigma) = - \int_{\underline{\theta}}^{b_r - s} [1 - F_c(s + \theta_i|\sigma)]^{N-1} f'_{\theta}(\theta_i) d\theta_i. \quad (3.5)$$

Then, the subcontractor's objective function (3.2) can be rewritten as

$$\pi(t_j^i|N, n, b_r) = \max_s (s - t_j^i) [1 - F_t(\sigma^{-1}(s))]^{n-1} Q(s|N, b_r). \quad (3.6)$$

The decreasing function  $Q(s|\cdot)$  reminds us of the objective function in Hansen (1988). He models the upstream procurement auction under which the winning bid negatively affects the quantity demanded in the downstream market. Since our  $Q(s|\cdot)$  can be interpreted as a demand schedule (the probability demanded), these two models are coincides. A non-trivial extension made by our model is that the downstream demand schedule is endogenously derived from the downstream auction model, while it is exogenously introduced in Hansen. Therefore, our model enables one to examine analytically to what extent environmental variables in the downstream auction, such as the number of bidders and the reservation price, affect the subcontractor's equilibrium strategy in upstream auctions. Furthermore, modeling the downstream competition allows one to examine what is the optimal mechanism in the downstream auction.

Now, as is done in Hansen, we verify that the subcontractor's strategy is indeed increasing. The following proposition claims that there exists a symmetric increasing equilibrium in the subcontractors' strategy.

**Proposition 1.** *The subcontractors' symmetric equilibrium strategy  $\sigma$  is strictly increasing in  $t$ . Hence, there exists a symmetric increasing equilibrium in the upstream auctions.*

*Proof.* Take derivative of (3.6) with respect to  $s$ , we have

$$[1 - F_t(t)]^{n-1}Q + (\sigma(t) - t) \times \left\{ -(n-1)f_t(t) \frac{1}{\sigma'(t)} [1 - F_t(t)]^{n-2}Q + [1 - F_t(t)]^{n-1} \frac{dQ}{ds} \right\} = 0.$$

Rearranging it gives

$$\sigma'(t|N, b_r) = (\sigma(t|N, b_r) - t)(n-1) \times \frac{f_t(t)}{1 - F_t(t)} \frac{Q(\sigma(t)|N, b_r)}{Q(\sigma(t)|N, b_r) + [\sigma(t_i) - t] \frac{dQ}{ds}}. \quad (3.7)$$

For any  $t \in [\underline{t}, \bar{t}]$ ,  $\sigma(t) \geq t$  holds with boundary condition  $\sigma(\bar{t}) = \bar{t}$ . Suppose there were an interval  $(t^-, t^+) \subset [\underline{t}, \bar{t})$  such that  $\sigma'(t^+) = 0$  and for all  $\hat{t} \in [t^-, t^+)$ ,  $\sigma'(\hat{t}) < 0$ . Then, from (3.7),  $\sigma(t^+) = t^+$ . First, we show that  $t^- \leq \underline{t}$ . Suppose  $t^-$  were strictly greater than  $\underline{t}$ . Then, we have  $\sigma'(t^-) = 0$ , implying that  $\sigma(t^-) = t^-$ . Since  $\sigma$  is decreasing at some  $t \in (t^-, t^+)$ ,  $\sigma(t) < t$  for all  $t \in (t^-, t^+)$ . So we have contradiction. Second, we show that  $t^+ \geq \bar{t}$ . Suppose it were not true. Then, because of the differentiability of  $\sigma$ , there should exist  $t \in (t^+, \bar{t}]$  such that  $\sigma(t) < t$ , which also contradicts the condition that  $\sigma(t) \geq t$ . Finally, we show that it is impossible for

$\sigma'$  to be negative throughout  $\hat{t} \in [\underline{t}, \bar{t}]$ . Suppose it were the case. Since  $Q'(s)$  is always negative, and  $Q$  is nonnegative, we must have  $Q(\sigma(\bar{t})) > 0$ . Since  $\sigma$  and  $Q$  are continuous and differentiable, there exists  $\tilde{t} < \bar{t}$  such that  $\lim_{t \downarrow \tilde{t}} Q(\sigma(t)) + (\sigma(t) - t)Q'(\sigma(t)) = 0$ . It entails that, for any  $t \in (\tilde{t}, s_t)$ ,  $Q(\sigma(t)) + (\sigma(t) - t)Q'(\sigma(t)) > 0$  implying that  $\sigma'(t) > 0$ . We reached a contradiction.  $\square$

To obtain the subcontractor's bidding function, we take derivative with respect to  $s$  on (3.6), and suppress sub- and superscripts. Then, we have the following first order condition

$$\begin{aligned} & t \left\{ [1 - F_{t(t)}]^{n-1} \frac{dQ}{d\sigma} \frac{d\sigma}{dt} - (n-1) f_t(t) [1 - F_{t(t)}]^{n-2} Q(\sigma(t)) \right\} \\ & = \sigma'(t) [1 - F_{t(t)}]^{n-1} Q(\sigma(t)) - \sigma(t) (n-1) f_t(t) [1 - F_{t(t)}]^{n-2} Q(\sigma(t)) \\ & \quad + \sigma(t) [1 - F_{t(t)}]^{n-1} \frac{dQ}{d\sigma} \frac{d\sigma}{dt}, \end{aligned}$$

where we replace  $s = \bar{\sigma}(t)$  and  $\bar{\sigma}^{-1}(s) = t$  due to  $\sigma$  being strictly increasing. This holds for any  $\hat{t} \in [t, \bar{t}]$  in equilibrium. Take integral from  $t$  through  $\bar{t}$  and use integral by parts on the right hand side, one obtains

$$\sigma(t|N, n, b_r) = t + \frac{\int_t^{\bar{t}} [1 - F_{t(\hat{t})}]^{n-1} Q(\sigma(\hat{t})|N, b_r) d\hat{t}}{[1 - F_{t(t)}]^{n-1} Q(\sigma(t)|N, b_r)}. \quad (3.8)$$

It is not necessary to obtain the explicit form of the subcontractor's bidding function to investigate whether their strategy is affected by the downstream competition. However, we need to make more detailed examination for the property of  $Q(\cdot)$ . The next lemma characterizes the subcontractor's equilibrium strategy  $\sigma$ .



**Lemma 1.** Suppose  $F_\theta$  and  $F_t$  are IHR. Then, for any  $t \in [\underline{t}, \bar{t}]$ ,  $\sigma(t|m, b_r)$  is strictly monotonic in a real number  $m \geq 1$

*Proof.* Replacing  $N$  with  $m \geq 1$ , one can rewrite (3.6) as

$$\pi(t_j^i|m, b_r) = \max_s (s - t_j^i) [1 - F_t(\sigma^{-1}(s))]^{n-1} Q(s|m, b_r).$$

The first order condition is given by

$$\frac{1}{\sigma(t|m, b_r) - t} = -\frac{Q'(\sigma(t|m, b_r)|m, b_r, \sigma)}{Q(\sigma(t|m, b_r)|m, b_r, \sigma)} + (n-1) \frac{f_t(t)}{1 - F_t(t)} \frac{1}{\sigma'(t|m, b_r)}.$$

Suppose by contrary that there exists  $t$  such that  $\xi(t) = \sigma(t|m, b_r) = \sigma(t|m', b_r)$  for some  $m' > m$ . Then, we have

$$\frac{1}{\sigma(t) - t} = -\frac{Q'(\sigma(t)|m, b_r)}{Q(\sigma(t)|m, b_r)} + (n-1) \frac{f_t(t)}{1 - F_t(t)} \frac{1}{\sigma'(t)}.$$

The right hand side is constant, whereas  $-\frac{Q'(\sigma(t|m, b_r)|m, b_r, \xi)}{Q(\sigma(t|m, b_r)|m, b_r, \xi)}$  is strictly increasing in  $m$ .<sup>44</sup> Therefore, we reached a contradiction. Hence,  $\sigma(t|N, b_r)$  is strictly monotonic in  $N$ . □

<sup>44</sup>Suppose  $F_\theta$  and  $F_t$  have monotone increasing hazard rate (IHR). Then,  $-\frac{Q'(s|m, b_r, \sigma)}{Q(s|N, b_r, \sigma)}$  is strictly increasing in  $m$  and weakly decreasing in  $-b_r$  provided that  $\sigma(t)$  is independent from  $m$  and  $b_r$ .

*Proof.* By (3.4) and (3.5), we have

$$-\frac{Q'(s|m, b_r)}{Q(s|m, b_r)} = \frac{\int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \theta_i)]^{m-1} f'_\theta(\theta_i) d\theta_i}{\int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \theta_i)]^{m-1} f_\theta(\theta_i) d\theta_i} \quad (3.9)$$

Let  $k_0$  be a positive number such that  $\frac{f'_\theta(\underline{\theta})}{f_\theta(\underline{\theta})} = k_0$ . Since  $\frac{f'_\theta(\theta)}{f_\theta(\theta)}$  is non-increasing in  $\theta$ , there exists a non-negative and non-increasing function  $k(\theta)$  such that for all  $\theta \in [\underline{\theta}, b_r - s]$

$$\frac{f'_\theta(\theta)}{f_\theta(\theta)} - k_0 = k(\theta),$$

**Lemma 2.** Suppose  $F_\theta$  and  $F_t$  are IHR. Then, for any  $t \in [\underline{t}, \bar{t}]$   $\sigma(t|N, b_r)$  is weakly monotonic in  $b_r$ .

*Proof.* Let  $b'_r > b_r$ . Then, one obtains  $\sigma(b_r|m, b'_r) > \sigma(b_r|m, b_r)$  since  $\sigma(b'_r|m, b'_r) = b'_r$  and  $\sigma(b_r|m, b_r) = b_r$  by the boundary condition. Also, one obtains  $-\frac{Q'(\cdot|m, b'_r)}{Q(\cdot|m, b'_r)} <$  with  $k(\underline{\theta}) = 0$ . So, we have  $f'_\theta(\theta) = [k_0 + k(\theta)]f_\theta(\theta)$ . Substituting it into (3.9) gives

$$\begin{aligned} -\frac{Q'(s|m, b_r)}{Q(s|m, b_r)} &= k_0 + \left[ \frac{\int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \theta_i)]^{m-1} k(\theta_i) f_\theta(\theta_i) d\theta_i}{\int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \theta_i)]^{m-1} f_\theta(\theta_i) d\theta_i} \right] \\ &= k_0 + \int_{\underline{\theta}}^{b_r-s} g(\theta_i|m) k(\theta_i) d\theta_i, \end{aligned} \quad (3.10)$$

where  $g(\theta|m, b_r) \equiv [1 - F_c(s + \theta)]^{m-1} f_\theta(\theta) / \int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \theta_i)]^{m-1} f_\theta(\theta_i) d\theta_i$ . Define  $G(\theta|m, b_r) = \int_{\underline{\theta}=\theta}^{\theta} g(\hat{\theta}|\cdot) d\hat{\theta}$ . Then, for any  $m' > m$  one obtains

$$\begin{aligned} &G(\theta|m') - G(\theta|m) \\ &= \frac{\int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m'-1} f_\theta(\hat{\theta}) d\hat{\theta}}{\int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \tilde{\theta})]^{m'-1} f_\theta(\tilde{\theta}) d\tilde{\theta}} - \frac{\int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m-1} f_\theta(\hat{\theta}) d\hat{\theta}}{\int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \tilde{\theta})]^{m-1} f_\theta(\tilde{\theta}) d\tilde{\theta}} \\ &= \frac{1}{\Delta} \left\{ \int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m'-1} f_\theta(\hat{\theta}) d\hat{\theta} \int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \tilde{\theta})]^{m-1} f_\theta(\tilde{\theta}) d\tilde{\theta} \right. \\ &\quad \left. - \int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m-1} f_\theta(\hat{\theta}) d\hat{\theta} \int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \tilde{\theta})]^{m'-1} f_\theta(\tilde{\theta}) d\tilde{\theta} \right\} \\ &= \frac{1}{\Delta} \left\{ \int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m'-1} f_\theta(\hat{\theta}) d\hat{\theta} \int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \tilde{\theta})]^{m-1} f_\theta(\tilde{\theta}) d\tilde{\theta} \right. \\ &\quad \left. - \int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m-1} f_\theta(\hat{\theta}) d\hat{\theta} \int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \tilde{\theta})]^{m'-1} f_\theta(\tilde{\theta}) d\tilde{\theta} \right\} \end{aligned}$$

By mean value theorem, one obtains

$$\begin{aligned} &= \frac{1}{\Delta} \left\{ [(1 - F_c(\theta^-))^{m'-m} - (1 - F_c(\theta^+))^{m'-m}] \right. \\ &\quad \left. \times \int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m-1} f_\theta(\hat{\theta}) d\hat{\theta} \int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \tilde{\theta})]^{m-1} f_\theta(\tilde{\theta}) d\tilde{\theta} \right\}, \end{aligned}$$

where  $\theta^- \in [\underline{\theta}, \theta]$  and  $\theta^+ \in [\theta, b_r - s]$ . Since  $F_c$  is strictly increasing, the whole terms are strictly positive. Hence, given an increasing function  $\sigma(t)$  independent from  $m$ ,  $G(\theta|m')$  is first-order stochastically dominated by  $G(\theta|m)$ . Since  $k(\theta_i)$  is non-increasing,  $-Q'(s|m, b_r)/Q(s|m, b_r)$  is increasing in  $m$  if  $\sigma$  is independent from  $m$ .  $\square$

$-\frac{Q'(\cdot|m, b_r)}{Q(\cdot|m, b_r)}$  since  $-\frac{Q'}{Q}$  is (weakly) decreasing in  $b_r$ .<sup>45</sup> Suppose by contradiction there exists  $t < b_r$  such that  $\sigma(t|m, b'_r) < \sigma(t|m, b_r)$ . Since  $\sigma(b_r|m, b'_r) > \sigma(b_r|m, b_r)$ , there must exist  $\tilde{t} \in (t, b_r)$  such that  $\sigma(\tilde{t}|m, b'_r)$  crosses  $\sigma(\tilde{t}|m, b_r)$  from below. Now, let  $\sigma(\tilde{t}) \equiv \sigma(\tilde{t}|m, b'_r) = \sigma(\tilde{t}|m, b_r)$ . Then, from (3.11) one obtains

$$\begin{aligned} & -\frac{Q'(\sigma(\tilde{t})|m, b'_r, \sigma)}{Q(\sigma(\tilde{t})|m, b'_r, \sigma)} + (n-1) \frac{f_t(\tilde{t})}{1 - F_t(\tilde{t})} \frac{1}{\sigma'(\tilde{t}|m, b'_r)} \\ & = -\frac{Q'(\sigma(\tilde{t})|m, b_r, \sigma)}{Q(\sigma(\tilde{t})|m, b_r, \sigma)} + (n-1) \frac{f_t(\tilde{t})}{1 - F_t(\tilde{t})} \frac{1}{\sigma'(\tilde{t}|m, b_r)}. \end{aligned}$$

<sup>45</sup>Let  $b'_r > b_r$ . From equation (3.10) one obtains

$$\begin{aligned} & G(\theta|m, b'_r) - G(\theta|m, b_r) \\ & = \frac{\int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m-1} f_{\theta}(\hat{\theta}) d\hat{\theta}}{\int_{\underline{\theta}}^{b'_r-s} [1 - F_c(s + \tilde{\theta})]^{m-1} f_{\theta}(\tilde{\theta}) d\tilde{\theta}} - \frac{\int_{\underline{\theta}}^{\theta} [1 - F_c(s + \hat{\theta})]^{m-1} f_{\theta}(\hat{\theta}) d\hat{\theta}}{\int_{\underline{\theta}}^{b_r-s} [1 - F_c(s + \tilde{\theta})]^{m-1} f_{\theta}(\tilde{\theta}) d\tilde{\theta}} \\ & \leq 0. \end{aligned}$$

This implies that  $G(\theta|m, b'_r)$  first-order stochastically dominates  $G(\theta|m, b_r)$  and that equality holds if and only if  $F_c(b_r) = 1$ . Hence, from equation (3.10) one obtains

$$\begin{aligned} & -\frac{Q'(s|m, b'_r)}{Q(s|m, b'_r)} - \left( -\frac{Q'(s|m, b_r)}{Q(s|m, b_r)} \right) \\ & = \int_{\underline{\theta}}^{b'_r-s} g(\theta_i|m, b'_r) k(\theta_i) d\theta_i - \int_{\underline{\theta}}^{b_r-s} g(\theta_i|m, b_r) k(\theta_i) d\theta_i \\ & = \int_{\underline{\theta}}^{b_r-s} [g(\theta_i|m, b'_r) - g(\theta_i|m, b_r)] k(\theta_i) d\theta_i + \int_{b_r-s}^{b'_r-s} g(\theta_i|m, b'_r) k(\theta_i) d\theta_i \\ & = k(\theta^-) \int_{\underline{\theta}}^{b_r-s} [g(\theta_i|m, b'_r) - g(\theta_i|m, b_r)] d\theta_i + k(\theta^+) \int_{b_r-s}^{b'_r-s} g(\theta_i|m, b'_r) d\theta_i \\ & \leq 0, \end{aligned}$$

where  $\theta^- < \theta^+$ . The third equality is obtained by mean value theorem. Furthermore,  $\int_{\underline{\theta}}^{b'_r-s} g(\cdot|m, b'_r) = \int_{\underline{\theta}}^{b_r-s} g(\cdot|m, b_r) = 1$ , or equivalently

$$\int_{\underline{\theta}}^{b_r-s} [g(\theta|m, b'_r) - g(\theta|m, b_r)] d\theta + \int_{\underline{\theta}}^{b'_r-s} g(\theta|m, b'_r) d\theta = 0.$$

Together with the fact that  $k(\cdot)$  is non-increasing, one obtains the last inequality.

Therefore,  $\sigma'(\tilde{t}|m, b'_r) < \sigma'(\tilde{t}|m, b_r)$  must hold. This implies that if two bidding functions cross at  $\tilde{t}$ , then  $\sigma(t|m, b'_r)$  crosses  $\sigma(t|m, b_r)$  from above. So, one obtains a contradiction.  $\square$

Using these two lemmas it is shown that prime contractors have a stronger bargaining power against subcontractors as the number of prime contractors increases or reservation price declines.

**Proposition 2.** *Subcontractors bid more aggressively in upstream auctions as the number of prime contractors rises or the reservation price drops in the procurement auction.*

*Proof.* Define a strictly increasing function  $\hat{\sigma}(t)$ , which satisfies the boundary condition  $\hat{\sigma}(\bar{t}) = \bar{t}$ . Then, we have

$$\hat{\sigma}(t) - \sigma(t) = \int_t^{\bar{t}} [\sigma'(\tau) - \hat{\sigma}'(\tau)] d\tau \quad (3.11)$$

Suppose  $\hat{\sigma}(t) > \sigma(t)$  for any  $t \in [\underline{t}, \bar{t})$ . Then, (3.12) and the boundary condition,  $\hat{\sigma}(\bar{t}) = \sigma(\bar{t})$  ensure that there exists  $\tilde{t} \in [\underline{t}, \bar{t})$  such that  $\sigma'(\tilde{t}) > \hat{\sigma}'(\tilde{t})$ . Therefore, we have a set of  $\tilde{t}$  such that

$$\hat{\sigma}(\tilde{t}) > \sigma(\tilde{t}) > 0 \quad \text{and} \quad \sigma'(\tilde{t}) > \hat{\sigma}'(\tilde{t}) > 0. \quad (3.12)$$

Now, suppose by contradiction that  $\sigma(t|N, b_r)$  is strictly increasing in  $N$  for any  $t < \bar{t}$ . Then, there exists  $\tilde{t} < \bar{t}$  such that  $\sigma(\tilde{t}|N, b_r) > \sigma(\tilde{t}|N + 1, b_r) > 0$  and

$\sigma'(\tilde{t}|N, b_r) > \sigma'(\tilde{t}|N + 1, b_r) > 0$ . Hence, one obtains

$$\begin{aligned}
& - \frac{Q'(\sigma(\tilde{t}|N + 1, b_r)|N + 1, b_r, \sigma)}{Q(\sigma(\tilde{t}|N + 1, b_r)|N + 1, b_r, \sigma)} \\
& = \frac{1}{\sigma(\tilde{t}|N + 1, b_r) - \tilde{t}} - (n - 1) \frac{f_t(\tilde{t})}{1 - F_t(\tilde{t})} \frac{1}{\sigma'(\tilde{t}|N + 1, b_r)} \\
& < \frac{1}{\sigma(\tilde{t}|N, b_r) - \tilde{t}} - (n - 1) \frac{f_t(\tilde{t})}{1 - F_t(\tilde{t})} \frac{1}{\sigma'(\tilde{t}|N, b_r)} \\
& = - \frac{Q'(\sigma(\tilde{t})|N, b_r, \sigma)}{Q(\sigma(\tilde{t})|N, b_r, \sigma)} \tag{3.13}
\end{aligned}$$

for some  $\tilde{t}$ . This must hold if  $N = 1$  and  $b_r$  is large enough so that  $b_r - \tilde{t} > \bar{\theta}$ .

Then, from (3.4)

$$Q(s|m, b_r, \sigma) = \int_{\underline{\theta}}^{b_r - s} f_{\theta}(\theta_i) d\theta_i = 1,$$

and  $Q'(s|m, b_r, \sigma) = 0$ . So,  $-\frac{Q'(\sigma(t|N, b_r)|N, b_r, \sigma)}{Q(\sigma(t|N, b_r)|N, b_r, \sigma)}$  is zero if  $N = 1$ , whereas it is strictly positive if  $N > 1$ . This contradicts (3.13). Thus,  $\sigma(t|N + 1, b_r)$  is strictly decreasing in  $N$ . We have already shown that  $\sigma(t|N, b_r)$  is weakly increasing in  $b_r$  in Lemma 5. □

Note that subcontractors are squeezed only if the first-price auction is used in upstream auctions. Subcontractors will report their costs truthfully in upstream auctions if the second-price auction is employed by prime contractors.

### 3.4 The downstream auction

It also follows that the distribution of the bidder's private information is endogenously determined by the number of bidder  $N$  and reservation price  $b_r$ ; Unlike

the standard auction model where the bidder's private information  $F_c$  is independent from  $N$  and  $b_r$ , the auction including pre-award subcontract competition entails that the bidder's cost distribution  $F_c$  is endogenously determined by the competitive environment of the auction.

Hence, we have the following proposition

**Proposition 3.** *In the downstream auction, the distribution of the bidders' private information is given endogenously by the number of bidders and reservation price, namely  $F_c(c_i|N, b_r)$ . Furthermore, the distribution shifts in the sense of first-order stochastic dominance i.e., for some  $c_i$ ,  $F_c(c_i|N, b_r) > F_c(c_i|N+1, b_r)$  for any positive integer  $N$  and  $F_c(c_i|N, b_r) > F_c(c_i|N, b'_r)$  for any  $b'_r < b_r$ .*

*Proof.* The cumulative distribution function of the prime contractor's cost,  $c$ , is given by

$$F_c(c|N, b_r) = F_t(\sigma^{-1}(c|N, b_r)).$$

For any  $t$ ,  $\sigma(t)$  rises as  $N$  decreases or  $b_r$  increases. Equivalently, for any  $s$ ,  $\sigma^{-1}(s|N, b_r)$  declines as  $N$  decreases or  $b_r$  increases. Hence,  $F_t(\sigma^{-1}(c|N+1, b_r))$  is first-order stochastically dominated by  $F_t(\sigma^{-1}(c|N, b_r))$ . Similarly, for any  $b_r < b'_r$ ,  $F_t(\sigma^{-1}(c|N, b'_r))$  is first-order stochastically dominated by  $F_t(\sigma^{-1}(c|N, b_r))$ .  $\square$

Hansen suggests that endogeneity of quantity reduces bids under the first-price rule, whereas the optimal strategy is unchanged in the second-price rule. Similar arguments hold in upstream subcontract auctions.

**Proposition 4.** *The expected lowest subcontract bid accepted by each prime contractor is greater if the second-price auction is used in upstream competitions.*

*Proof.* Revenue equivalence does not hold in upstream auctions.  $\square$

**Proposition 5.** *The prime contractor's ex ante expected payoff depends on the number of her rival  $N - 1$  and reservation price.*

*Proof.*  $\theta = 0$  The objective function of the prime contractor who selects a subcontract with the first price subcontract auction is given by

$$u(c_i|N, b_r) = \max_b (b - c_i) [1 - F_c(\beta^{-1}(b)|N)]^{N-1},$$

for any  $c_i \leq b_r$  with boundary condition  $u(b_r) = 0$ . From the envelope integral formula, we have

$$\begin{aligned} u(c(t_{(1)}^i)|N, b_r) &= \int_{c(t_{(1)}^i)}^{b_r} [1 - F_c(\hat{c}|N)]^{N-1} d\hat{c} \\ &= \int_{t_{(1)}^i}^{b_r} [1 - F_c(\hat{c}|N)]^{N-1} d\hat{c} - \int_{t_{(1)}^i}^{c(t_{(1)}^i)} [1 - F_c(\hat{c}|N)]^{N-1} d\hat{c}. \end{aligned}$$

On the other hand, the objective function of the prime contractor who selects a subcontract with the second price subcontract auction and whose opponents select their subcontract with the first price subcontract auctions is given by

$$u^{SP}(c_i|N, b_r) = \int_{t_{(1)}^i}^{b_r} \max_{b|t_{(2)}^i} (b - t_{(2)}^i) [1 - F_c(\beta^{-1}(b)|N)]^{N-1} \frac{(n-1)f_t(t_{(2)}^i) [1 - F_t(t_{(2)}^i)]^{n-2}}{[1 - F_t(t_{(1)}^i)]^{n-1}} dt_{(2)}^i$$

Since  $b$  is the maximizer for any  $t_{(2)}^i$ , one must have

$$\frac{dw(t_{(2)}^i)}{dt_{(2)}^i} = - [1 - F_c(t_{(2)}^i)]^{N-1},$$

where  $w(\cdot) = (\beta(c(t_{(2)}^i)) - t_{(2)}^i) [1 - F_c(t_{(2)}^i)]^{N-1}$ . Since this holds for any  $\hat{t} \in [t_{(2)}^i, b_r]$ , one obtains

$$(\beta(c(t_{(2)}^i)) - t_{(2)}^i) [1 - F_c(t_{(2)}^i)]^{N-1} = \int_{t_{(2)}^i}^{b_r} [1 - F_c(\hat{t})]^{N-1} d\hat{t}$$

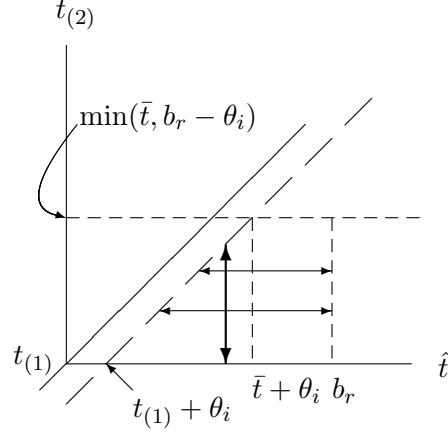
Plug it back to (3.15), one obtains

$$\begin{aligned} u^{SP}(c_i|N, b_r) &= \int_{t_{(1)}^i}^{b_r} \int_{t_{(2)}^i}^{b_r} [1 - F_c(\hat{t})]^{N-1} d\hat{t} \frac{(n-1)f_t(t_{(2)}^i) [1 - F_t(t_{(2)}^i)]^{n-2}}{[1 - F_t(t_{(1)}^i)]^{n-1}} dt_{(2)}^i \\ &= \int_{\hat{t}=t_{(1)}^i}^{b_r} \int_{t_{(2)}^i=t_{(1)}^i}^{\hat{t}} [1 - F_c(\hat{t})]^{N-1} \frac{(n-1)f_t(t_{(2)}^i) [1 - F_t(t_{(2)}^i)]^{n-2}}{[1 - F_t(t_{(1)}^i)]^{n-1}} dt_{(2)}^i d\hat{t} \\ &= \int_{\hat{t}=t_{(1)}^i}^{b_r} [1 - F_c(\hat{t})]^{N-1} d\hat{t} - \int_{\hat{t}=t_{(1)}^i}^{b_r} \left( \frac{1 - F_t(\hat{t})}{1 - F_t(t_{(1)}^i)} \right)^{n-1} [1 - F_c(\hat{t})]^{N-1} d\hat{t} \end{aligned}$$

$c(t_{(1)}^i) = E(\min\{\min_{j \neq i}\{t_{(1)}^j\}, t_{(2)}^i\} | t_{(1)}^i) \leq E(t_{(2)}^i | t_{(1)}^i)$ . Hence,

$$\begin{aligned} &\int_{t_{(1)}^i}^{c(t_{(1)}^i)} [1 - F_c(c)]^{N-1} dc \\ &\leq [1 - F_c(t_{(1)}^i)]^{N-1} (c(t_{(1)}^i) - t_{(1)}^i) \\ &= [1 - F_c(t_{(1)}^i)]^{N-1} \int_{\hat{t}=t_{(1)}^i}^{\bar{t}} \left( \frac{1 - F_t(\hat{t})}{1 - F_t(t_{(1)}^i)} \right)^{n-1} \left[ \frac{1 - F_c(c(\hat{t}))}{1 - F_c(c(t_{(1)}^i))} \right]^{N-1} d\hat{t} \\ &\leq [1 - F_c(t_{(1)}^i)]^{N-1} \int_{\hat{t}=t_{(1)}^i}^{\bar{t}} \left( \frac{1 - F_t(\hat{t})}{1 - F_t(t_{(1)}^i)} \right)^{n-1} \left[ \frac{1 - F_c(\hat{t})}{1 - F_c(t_{(1)}^i)} \right]^{N-1} d\hat{t} \\ &= \int_{\hat{t}=t_{(1)}^i}^{\bar{t}} \left( \frac{1 - F_t(\hat{t})}{1 - F_t(t_{(1)}^i)} \right)^{n-1} [1 - F_c(\hat{t})]^{N-1} d\hat{t} \end{aligned}$$





$$\begin{aligned} \frac{d}{dt} \frac{F_t(t)}{F_t(\sigma(t))} &= \frac{f_t(t)F_t(\sigma(t)) - F_t(t)f_t(\sigma(t)) \cdot \sigma'(t)}{(F_t(\sigma(t)))^2} \\ &= \frac{F_t(t)}{F_t(\sigma(t))} \left\{ \frac{f_t(t)}{F_t(t)} - \frac{f_t(\sigma(t)) \cdot \sigma'(t)}{F_t(\sigma(t))} \right\} \geq 0. \end{aligned}$$

The last inequality holds since  $\frac{f_t(\cdot)}{F_t(\cdot)}$  is non-increasing,  $\sigma(t) \geq t$  and  $\sigma' \leq 1$ . Hence,  $\frac{F_t(t)}{F_t(\sigma(t))}$  is increasing in  $t$ .

$\theta = r.v$ . The objective function of the prime contractor who selects a subcontract with the first price subcontract auction is given by

$$u(t_{(1)}^i, \theta_i | N, b_r) = \max_b (b - c(t_{(1)}^i)) [1 - F_c(\beta^{-1}(b) | N)]^{N-1},$$

for any  $c_i \leq b_r$  with boundary condition  $u(b_r) = 0$ . From the envelope integral formula, we have

$$\begin{aligned} u(c(t_{(1)}^i), \theta_i | N, b_r) &= \int_{\sigma(t_{(1)}^i) + \theta_i}^{b_r} [1 - F_c(\hat{c} | N)]^{N-1} d\hat{c} \\ &= \int_{t_{(1)}^i + \theta_i}^{b_r} [1 - F_c(\hat{c} | N)]^{N-1} d\hat{c} - \int_{t_{(1)}^i + \theta_i}^{\sigma(t_{(1)}^i) + \theta_i} [1 - F_c(\hat{c} | N)]^{N-1} d\hat{c}. \end{aligned}$$

On the other hand, the objective function of the prime contractor who selects a subcontract with the second price subcontract auction and whose opponents select their subcontract with the first price subcontract auctions is given by

$$u^{SP}(t_{(1)}^i, \theta_i | N, b_r) = \int_{t_{(1)}^i}^{b_r} \max_{b | t_{(2)}^i, \theta_i} (b - t_{(2)}^i - \theta_i) \cdot [1 - F_c(\beta^{-1}(b) | N)]^{N-1} \frac{(n-1) f_t(t_{(2)}^i) [1 - F_t(t_{(2)}^i)]^{n-2}}{[1 - F_t(t_{(1)}^i)]^{n-1}} dt_{(2)}^i \quad (3.15)$$

Since  $b$  is the maximizer for any  $t_{(2)}^i + \theta_i$ , one must have

$$\frac{dw(t_{(2)}^i)}{d(t_{(2)}^i + \theta_i)} = - [1 - F_c(t_{(2)}^i + \theta_i)]^{N-1},$$

where  $w(\cdot) = (\beta(\sigma(t_{(2)}^i) + \theta_i) - (t_{(2)}^i + \theta_i)) [1 - F_c(t_{(2)}^i + \theta_i)]^{N-1}$ . Since this holds for any  $\hat{t} \in [t_{(2)}^i, b_r]$ , one obtains

$$(\beta(c(t_{(2)}^i)) - t_{(2)}^i) [1 - F_c(t_{(2)}^i + \theta_i)]^{N-1} = \int_{t_{(2)}^i + \theta_i}^{b_r} [1 - F_c(\hat{c})]^{N-1} d\hat{c}$$

Plug it back to (3.15), one obtains

$$\begin{aligned}
& u^{SP}(t_{(1)}^i, \theta_i | N, b_r) \\
&= \int_{t_{(2)}^i = t_{(1)}^i}^{\min\{\bar{t}, b_r - \theta\}} \int_{\hat{c} = t_{(2)}^i + \theta_i}^{b_r} [1 - F_c(\hat{c})]^{N-1} d\hat{c} \frac{(n-1) f_t(t_{(2)}^i) [1 - F_t(t_{(2)}^i)]^{n-2}}{[1 - F_t(t_{(1)}^i)]^{n-1}} dt_{(2)}^i \\
&= \int_{\hat{c} = t_{(1)}^i + \theta_i}^{\min\{b_r, \bar{t} + \theta_i\}} \int_{t_{(2)}^i = t_{(1)}^i}^{\hat{c} - \theta_i} [1 - F_c(\hat{c})]^{N-1} \frac{(n-1) f_t(t_{(2)}^i) [1 - F_t(t_{(2)}^i)]^{n-2}}{[1 - F_t(t_{(1)}^i)]^{n-1}} dt_{(2)}^i d\hat{c} \\
&\quad + \mathbf{1}_{\{\bar{t} + \theta_i < b_r\}} \int_{\hat{c} = \bar{t} + \theta_i}^{b_r} \int_{t_{(2)}^i = t_{(1)}^i}^{\bar{t}} [1 - F_c(\hat{c})]^{N-1} \frac{(n-1) f_t(t_{(2)}^i) [1 - F_t(t_{(2)}^i)]^{n-2}}{[1 - F_t(t_{(1)}^i)]^{n-1}} dt_{(2)}^i d\hat{c} \\
&= \int_{\hat{c} = t_{(1)}^i + \theta_i}^{b_r} [1 - F_c(\hat{c})]^{N-1} d\hat{c} - \int_{\hat{c} = t_{(1)}^i + \theta_i}^{\bar{t} + \theta_i} \left( \frac{1 - F_t(\hat{c} - \theta_i)}{1 - F_t(t_{(1)}^i)} \right)^{n-1} [1 - F_c(\hat{c})]^{N-1} d\hat{c} \\
&= \int_{\hat{c} = t_{(1)}^i + \theta_i}^{b_r} [1 - F_c(\hat{c})]^{N-1} d\hat{c} - \int_{\hat{t} = t_{(1)}^i}^{\bar{t}} \left( \frac{1 - F_t(\hat{t})}{1 - F_t(t_{(1)}^i)} \right)^{n-1} [1 - F_c(\hat{t} + \theta_i)]^{N-1} d\hat{t}
\end{aligned}$$

Since  $F_c$  is differentiable, the Taylor expansion gives

$$\begin{aligned}
& \int_{t_{(1)}^i + \theta_i}^{\sigma(t_{(1)}^i) + \theta_i} [1 - F_c(\hat{c})]^{N-1} d\hat{c} \\
&= [1 - F_c(t_{(1)}^i + \theta_i)]^{N-1} (\sigma(t_{(1)}^i) - t_{(1)}^i) \\
&\quad - \frac{N-1}{2!} [1 - F_c(t_{(1)}^i)]^{N-2} f_c(t_{(1)}^i) (\sigma(t_{(1)}^i) - t_{(1)}^i)^2 (t_{(1)}^i + \theta_i + \rho(\sigma(t_{(1)}^i) - t_{(1)}^i)) \\
&\leq [1 - F_c(t_{(1)}^i + \theta_i)]^{N-1} (\sigma(t_{(1)}^i) - t_{(1)}^i) \\
&= \int_{\hat{t} = t_{(1)}^i}^{\bar{t}} \left( \frac{1 - F_t(\hat{t})}{1 - F_t(t_{(1)}^i)} \right)^{n-1} [1 - F_c(\hat{t} + \theta_i)]^{N-1} d\hat{t} \\
&\leq \int_{\hat{t} = t_{(1)}^i}^{\bar{t}} \left( \frac{1 - F_t(\hat{t})}{1 - F_t(t_{(1)}^i)} \right)^{n-1} [1 - F_c(\hat{t} + \theta_i)]^{N-1} d\hat{t}
\end{aligned}$$

□

Whether do the prime contractors have an incentive to have the second-price rule to select subcontract bids?

Subcontractors bid more aggressively as increasing competition among their main contractors only in the case where the first-price rule is chosen by the prime contractor.

**Proposition 6.** *Suppose  $n = 1$ . Then, no prime contractor has an incentive to use the second-price auction to select subcontractors.*

*Proof.* Suppose  $n = 1$  and prime contractors set reservation price equal to  $\bar{t}$  in upstream auctions. Then, the subcontract price is always equal to  $\bar{t}$  if the second-price auction is used. On the other hand, the expected subcontract bid would equal  $E[\sigma(t|\cdot)] \leq \bar{t}$  if the first-price auction is used in upstream auctions.  $\square$

So we have the following corollary about the efficiency argument.

**Corollary 1.** *SP is likely to lead to an inefficient allocation, i.e., the subcontractor who is not lowest is picked more frequently.*

If the first-price auction is used, the subcontract payment described in the pre-award subcontract agreement is likely to be too high for the prime contractor to be cost disadvantaged in the downstream auction. It is true for the case where the subcontractor is the most efficient supplier among all the subcontractors including those of other primes. That is, the second-price auction is likely to induce the *ex post* inefficient allocation in subcontract.

### 3.4.1 The theory of optimal reservation price

**Proposition 7.** *Suppose the distribution function of the bidder's private signal is a function of the number of bidders. Then, the optimal reservation price will be dependent upon the number of participants in the auction.*

*Proof.* The expected return to the procurement buyer is given by

$$V [1 - (1 - F_c(b_r|N, b_r))^N] - N \int_{\underline{c}}^{b_r} [\hat{c} f_c(\hat{c}|N, b_r) + F_c(\hat{c}|N, b_r)] [1 - F_c(\hat{c}|N, b_r)]^{N-1} d\hat{c}$$

Taking derivative with respect to  $b_r$  gives the optimal reservation price  $b_r^*$

$$N [V f_c(b_r^*|N, b_r^*) - b_r^* f_c(b_r^*|N, b_r^*) - F_c(b_r^*|N, b_r^*)] [1 - F_c(b_r^*|N, b_r^*)]^{N-1} = 0$$

Hence, the expected winning bid is minimized if the procurement buyer sets the reservation price  $b_r^*$ , where

$$b_r^* = V - \frac{F_c(b_r^*|N, b_r^*)}{f_c(b_r^*|N, b_r^*)}.$$

□

### 3.4.2 The theory of auctions with endogenous participation

In the first stage, potential prime contractors decide their entry simultaneously. Then, in the second stage, each prime contractor who has decided to participate incurs a fixed amount of participation cost  $e$ , observes the number of actual prime contractors  $N$  in the procurement auction, draws her own signal  $t_i$ , and selects the

lowest price quotes offered by her potential subcontractors.<sup>46</sup> Finally, in the third stage, prime contractors submit a bid in the procurement auction.

**Proposition 8.** *A positive entry subsidy to participants (prime contractors) is revenue enhancing if participation is endogenously determined and the contract includes subcontracts.*

*Proof.*

$$\begin{aligned}
U(N+1) &= \int_c F_c(c|N+1) [1 - F_c(c|N+1)]^N dc \\
&= \int_c [1 - F_c(c|N+1)]^N dc - \int_c [1 - F_c(c|N+1)]^{N+1} dc \\
&= \int_c cN f_c [1 - F_c(c|N+1)]^{N-1} dc - \int_c c(N+1) f_c [1 - F_c(c|N+1)]^N dc \\
&= E(c_{1:N} | F_c(\cdot|N+1)) - E(c_{1:N+1} | F_c(\cdot|N+1)) \\
&< E(c_{1:N} | F_c(\cdot|N)) - E(c_{1:N+1} | F_c(\cdot|N+1)).
\end{aligned}$$

The last inequality holds due to the fact that  $E(c_{1:N} | F_c(\cdot|N)) > E(c_{1:N} | F_c(\cdot|N+1))$ .

Let  $S_1$  be the sum of expected gains obtained by both the auctioneer and  $N$  prime contractors, which is given by

$$S_1(N) = V - E(c_{1:N} | F_c(\cdot|N)) - Ne.$$

Suppose  $\bar{N}$  potential prime contractors participate in the auction with probability equal to  $p^*$ . For  $p^*$  to be a symmetric mixed-strategy entry equilibrium,  $p^*$  must

<sup>46</sup>We note that the order of the events in the second stage is crucial in our model. If price quotes are selected before  $N$  is observed, then our theory about the endogenous private information does not hold. Furthermore, if  $N$  can be observed before the participation cost is incurred, quite a few bidders may exit from the auction, anticipating the lower or negative gain from participation.

satisfy

$$\sum_{N=0}^{\bar{N}-1} B_N^{\bar{N}-1}(p^*)U(N+1) = e,$$

where  $B_N^{\bar{N}-k}(p) \equiv \binom{\bar{N}-k}{N} p^N (1-p)^{\bar{N}-k-N}$ . The expected sum of gains of both two parties,  $E(S_1|p)$ , is then given by

$$\begin{aligned} S_1(p) &= V - \sum_{N=0}^{\bar{N}} B_N^{\bar{N}}(p) [E(c_{1:N}|F_{c(\cdot|N)}) - Ne] \\ &= V - \sum_{N=0}^{\bar{N}} B_N^{\bar{N}}(p) E(c_{1:N}|F_{c(\cdot|N)}) - Npe. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\partial S}{\partial p} &= \sum_{N=0}^{\bar{N}-1} B_N^{\bar{N}-1}(p) [E(c_{1:N}|F_{c(\cdot|N)}) - E(c_{N+1;N+1}|F_{c(\cdot|N+1)})] - e \\ &> \sum_{N=0}^{\bar{N}-1} B_N^{\bar{N}-1}(p)U(N+1) - e \\ &= 0. \end{aligned}$$

Any pure monetary transfer upon entry does not prevent the extraction of full rents from the prime contractors. Therefore, a positive entry subsidy to the participants, which induces more entry and extracts more rent from subcontractors, result in more revenue from the auction.  $\square$

In symmetric independent value auctions, the bidder's *ex ante* expected payoff is identical to the expected marginal contribution to reducing the expected minimum cost (Engelbrecht-Wiggans (1993)).

If there is one more additional bidder (prime contractor), his expected marginal contribution to lowering the expected minimum cost is exactly the same as the *ex ante* payoff of each bidder. With subcontracting, however, further entry has two effect; not only does it reduce the expected social cost for the project from the viewpoint of order statistics, but also shifts the distribution of the minimum cost downward, which also reduces the expected social cost.

On the other hand, the *ex ante* expected payoff fails to capture the second effect. That is, despite subcontracting, the expected amount each bidder earns from the auction is still identical to the marginal contribution to decreasing the expected minimum cost in the sense of order statistics. The society obtains larger gain from further entry, implying that the entry subsidy enhances the auctioneer's gain.

Another interpretation is given as follows. There are three parties in the entire game, i) the government (auctioneer), ii) the prime contractors, and iii) the subcontractors. Because of entry, prime contractors' rents will be equal to zero. On the other hand, subcontractors earn positive expected rents although it would be decreasing on average as the number of bidders in the downstream auction increases. Subsidizing entry of prime contractors enables the procurement buyers to extract more rents from subcontractors. Hence, free entry is insufficient from the viewpoint of the procurement buyer's welfare.

### 3.5 Discussion

For simplicity, our theory focuses on the symmetric case where the number of subbids solicited is identical for all prime contractors. It is easily shown that the results obtained so far are unchanged even in cases that some prime contractors



solicit more subbids than the others. Since it is natural to think that each prime contractor does not know the number of subcontract bids solicited by other prime contractors, the procurement auction still satisfies the symmetric assumption such that each bidder is *ex ante* the same.

Subcontractors typically bid a specific prime contractor. The major reason for the practice comes from the past difficulties in working with other prime contractors. The past difficulties in working also make prime contractors to reject low bids from some subcontractors. Working with subcontractors who are considered to be insufficiently qualified results in higher risk for prime contractors. So, subcontractors bid their favorite prime contractors first and, if time permitted, getting through to the rest (Dyer and Kagel (1996)).

Nevertheless, what if subcontractors bid to multiple prime contractors? The conclusion is our result is robust in such a case as long as there is a subcontractor submitting a bid only to a particular prime contractor. Consider first the situation where subcontractors submit multiple bids, but there exist a set of subcontractors who bid only once. To compete against the firms exclusively affiliated with a single prime contractor, the remaining subcontractors still have to bid aggressively. As the downstream competition becomes severe, the former bid aggressively, which induces the latter accordingly to bid more aggressively.

Subcontractors do not mind the downstream auctions in the extreme case where each subcontractor bids for all prime contractors. Even in such extreme case, the model shows that subcontractors bid more aggressively as the reservation price in the downstream auction drops since  $\frac{\partial \sigma}{\partial b_r} < 0$  still holds in Proposition 11, regardless of  $N$ , the number of bidders in the downstream auction.

The endogeneity of prime contractor's cost distribution is maintained in the more generalized case. Furthermore, if the subcontractors does not know whether the other subcontractors bid for multiple prime contractors, then the subcontract competition can still be described as a symmetric equilibrium in which players are *ex ante* the same.

### 3.6 Conclusion

Most of the auction literature implicitly assume that the players are the agents who send a message directly to the principal (auctioneer). In reality, and in particular in the procurement auctions, goods and services are produced by a team of firms (main and sub firms). The lower-tiered subcontractors and suppliers can be non-negligible players who also possess private information in the Bayesian game.

Taking into account the lower tiered producers and suppliers, it may be obvious that the intensity of competition in the downstream auction affects not only the primary contractor's profit, but also the subcontractor's profits. Their aggressive bidding in the upstream auction helps their prime contractor win in the downstream auctions.

The main contribution of this paper lies in formulating an auction model including the vertically related production system that can be seen in most of the industries. Our theory suggests that an additional entrant to the downstream auction results in the prime contractor's stronger bargaining power against her subcontractors. In other words, prime contractors have the cost distribution endogenously determined by the number of bidders and the reservation price in the downstream auction.

The application of the theory is wide spread from joint bidding or bid consortium. Even though member firms have a close and trustable relationship with each other, there still remains a room where each member owns private information, which create the possibility for each member firm to obtain a rent against the representative firm of the consortium.

Throughout, we rule out the *ex post* negotiation between a prime and subcontractors. Prime contractors may abuse the good faith efforts; Practically, even if a pre-bid price quote is received, the prime contractor is still able to negotiate a lower price from the subcontractor. Or, primes may solicit companies to perform irrelevant categories of work or in the irrelevant geographic area. However, as noted in the Availability and Disparity Study by Caltrans in 2007, the good faith efforts are critical to achieve successful subcontracting, which obviously leads to an efficient production and greater profit for the winning prime contractor. Hence, our model does not consider the *ex post* negotiation between sub- and prime contractors.

The incentives for contracting is not only to reduce costs. For instance, Marechal and Morand (2003) point out that subcontracting can reduce the risk of potential change orders.<sup>47</sup> Given the sheer volume of procurement, it is clear that more serious research and evaluation is needed to investigate the effect of subcontracting.

<sup>47</sup>The effect of such ex post changes on procurement contracts is thoroughly analyzed in Bajari and Tadelis (2001).

## CHAPTER 4

# EQUILIBRIA IN ASYMMETRIC AUCTIONS WITH ENTRY

### Abstract

Regarding optimal design in the private value environment, there is an unsolved tension between the literature for asymmetric auctions and auctions with endogenous participation; The former suggests that well-designed distortive mechanisms are optimal (revenue maximizing) assuming the bidding costs are negligible, while the latter insists that the mechanisms with free entry and no distortion are optimal provided that the potential bidders are *ex ante* symmetric.

This paper is the first attempt to reconcile the tension by establishing a model for asymmetric auctions with costly participation. The main findings are threefold; First, optimal outcome is possible if and only if the mechanism is *ex post* efficient. Second, without any participation control, a coordination problem is likely that only the *weak* bidders participate and the *strong* bidders stay out. Finally, there is an entry fee/subsidization scheme which, together with an *ex post* efficient mechanism, induces the optimal outcome as a unique equilibrium.

## 4.1 Introduction

In the high-valued asset or procurement auctions, the costs for preparing bids are typically non-trivial. The costs each bidder incurs prior to bidding range from the information acquisition costs to transportation costs, and even opportunity costs of awarding. Potential bidders who anticipate that bidding is unprofitable may hesitate to do so; therefore, designing a mechanism accounting for the bidder's endogenous participation is crucial for the auction to be successful. The model of auction with endogenous entry has received comprehensive analysis motivated by such practical situations faced by the auctioneer (*e.g.*, McAfee and McMillan (1987b), Engelbrecht-Wiggans (1993), Levin and Smith (1994), Kjerstad and Vagstad (2000), Ye (2004)). Furthermore, empirical studies of auctions with endogenous participation are growing following the development in the theoretical endogenous participation models.<sup>48</sup>

The notable insights presented in the existing literature, however, crucially depend on the assumption that all the potential bidders are *ex ante* identical. This strong assumption may result in analyses being restrictive. Consider the case, for example, where a limited number of bidders participate frequently, but there are many other potential bidders who rarely enter the auction. Should revenue maximizing auctioneers precommit to running a distortive auction to encourage entry by the one-shot customers? Or, should they give up promoting competition and simply set an entry fee to extract more surplus from the frequenters? What if it turns out that those one-shot customers are likely to have higher valuation for the item? Do more entries and stronger competition create higher revenue for the seller? The

<sup>48</sup>See *e.g.* Athey and Haile (2006).

existing studies provide an ambiguous prediction for the auctions with asymmetric potential bidders.<sup>49</sup>

In this research, we provide the first theoretical analysis for asymmetric auctions with endogenous participation. The model we establish is an extension of the model of auctions with costly participation, in which risk-neutral potential bidders randomize their participation in the auction. The bidders who actually enter incur a fixed participation cost and acquire their private information. In this formulation, we relax the symmetric assumption in the following two ways; First, we suppose that potential bidders consist of two groups, and that the bidders in each group participate in the auction with probability  $p_1$  for one group, and  $p_2$  for the other.<sup>50</sup> The equilibrium we focus on in this paper is the *type symmetric* mixed-strategy entry equilibrium which is constituted by a pair of probabilities  $(p_1, p_2)$ . Second, we consider that the potential bidders in one group may be stronger than those in the other group, *i.e.*, the value distribution of a group of bidders stochastically dominates that of the remaining bidders.<sup>51</sup>

It is shown that there is at least one, and typically multiple type-symmetric equilibria. In some cases, a group of *weak* potential bidders enter with positive probability and all the *strong* bidders stay out in equilibrium. We then demonstrate that, if the mechanism is *ex post* efficient, participation is always efficient in the

<sup>49</sup>The theoretical model provided by Pevnitskaya (2003a) characterizes an equilibrium with asymmetric bidders, assuming that different potential bidders have different risk attitudes. She finds that some bidders with being less risk-averse tend to participate in the auction more frequently than the other in equilibrium provided that each bidder's risk attitude is common knowledge. However, the equilibrium bidding function analyzed in her study is essentially symmetric.

<sup>50</sup>Although the model considers only two groups, it can be extended to the case of three or more groups.

<sup>51</sup>There can be many forms of asymmetry. Our model does not restrict the form of asymmetry to stochastic dominance.

sense that the expected marginal contribution of an additional participant to social surplus equals the marginal costs for participation. However, we also show that the “efficient entry” is not always optimal for society and the seller: due to the stochastic entry process, some equilibria are more likely to end up with “too many” or “too few” bidders. These coordination costs are another source of efficiency loss, which makes all the efficient entry equilibria be sub-optimal, except one. In the case of symmetric potential bidders, for example, a symmetric equilibrium creates the lowest revenue because of the highest coordination costs among equilibria.

Facing such a multiplicity problem, how does the auctioneer induce the optimal participation? Our model suggests that, by introducing participation fee/subsidization contingent on the realization of participation, the desired entry is always induced as a unique equilibrium, regardless of the form of asymmetry. Moreover, the transfer scheme enables auctioneers to extract the entire surplus generated by the auction. Hence, sellers can implement the optimal entry by using a simple auction (English or second price sealed-bid auction) with a well-chosen monetary transfer scheme.

These findings shed a new light on the literature of auctions with asymmetric bidders as well as auctions with endogenous participation; First, our results contradict the theorems for optimal design problem with a fixed number of bidders, which assert that a positive reservation price or some distortive allocation favoring a group of bidders improves revenue for the seller (*e.g.*, Riley and Samuelson (1981), Myerson (1981), Bulow and Roberts (1989), McAfee and McMillan (1989)). However, this argument ignores the point that the rent extraction from a group of potential bidders could depress their participation. Our results illustrate that *ex post* efficiency is essential for efficient entry, and thus for optimal outcome, taking

endogenous participation into account. It follows that any distortive mechanism entails the sub-optimal outcome in the asymmetric auctions with potential bidders.

Second, the optimality of *ex post* efficient mechanisms challenges the ranking theorems. In the asymmetric auctions, revenue ranking between first and second price mechanisms are generally ambiguous. This ambiguity, however, disappears, considering entry. We show that, by asymmetry, first price auctions may attract more or less bidders, but the resulting excessive or deficient entry inflicts more burden on the seller, who bears all the participation costs. This story can easily be extended to the case of asymmetric auctions with affiliated private value (APV). Since affiliation gives greater advantage to second price mechanisms, first price mechanisms are still dominated by second price mechanisms under the APV environment.

Third, our model extends the theoretical analysis of auctions with endogenous participation for several aspects. There are two groups of literature for auctions with costly participation, investigating either an asymmetric equilibrium (*e.g.*, McAfee and McMillan (1987b)) or a symmetric equilibrium (*e.g.*, Levin and Smith (1994)). We provide a general theory which analyzes both simultaneously. This enables us to obtain a ranking method for social surplus and revenue across equilibria. Furthermore, the theorem is robust in cases of heterogeneous potential bidders. The symmetry assumption is somewhat disturbing since it is violated if it is common knowledge that some potential bidders are even a little likely to have higher valuation for the item.

Finally, our results provide a theoretical background for the experimental analysis for auctions with endogenous participation. Pevnitskaya (2003b) observes an evidence in the laboratory experiments that the same subjects are more likely to



participate than the others. Since we show that the participation game is similar to a coordination game where there are multiple equilibria, such outcome is possible as an equilibrium.

The remaining part of this paper is organized as follows; In section 4.2 we describe the model. Discussion is provided in section 4.6. Section 4.7 gives the conclusion of this paper. Proofs are given in an appendix.

## 4.2 Model

Consider a risk-neutral seller auctions a single indivisible item to two groups of risk-neutral potential bidders with unit demand. Suppose there are  $N_\tau$  potential bidders in group  $\tau \in \{1, 2\}$ , and let  $\mathcal{N}_\tau \equiv \{1, \dots, N_\tau\}$  denote the index set of group  $\tau$  potential bidders.

The transaction is described as a two-stage game. In the second stage, an auction takes place with  $n = (n_1, n_2)$  actual bidders to allocate the item subject to an allocation rule set by the auctioneer. In the first stage, each potential bidder simultaneously makes her decision to incur a fixed participation cost  $c_\tau$  and enter the auction game.

Throughout this paper, we suppose private values, *i.e.*, that one buyer's signal does not affect the other's preferences. Each bidder draws its own signal denoted by  $\sigma$  which is, without loss of generality, uniformly distributed between 0 and 1. The value of the item for a bidder in group  $\tau$  is captured by the valuation function  $v_\tau : [0, 1] \rightarrow \mathfrak{R}_+$ , which is strictly increasing and continuously differentiable. Finally, the seller's value for the item is normalized to equal zero.

The auction game consists of a set  $B_\tau^i \in B$  of bids for each bidder, an allocation rule  $x(\cdot|n) : B \rightarrow [0, 1]^n$ , and a payment rule  $k(\cdot|n) : B \rightarrow T^n$ . If the  $i$ th bidder in group  $\tau$  chooses a bid  $b_\tau^i \in B_\tau^i$ , then, given a bid profile  $b = (b_1^1, \dots, b_1^{n_1}, b_2^1, \dots, b_2^{n_2})$ , the bidder obtains the item with probability equal  $x_\tau^i(b|n) \in [0, 1]$ , and makes the expected payment  $k_\tau^i(b|n)$  to the seller.

The entry game begins with the seller's announcement on the assignment rule  $\xi = (\xi_1, \xi_2)$  with  $\xi_\tau : \times_{n \in \mathcal{N}} \xi_\tau(b|n)$  and  $\xi_\tau(b|n) = (\xi_\tau^1(b|n), \dots, \xi_\tau^{n_\tau}(b|n))$ , the payment rule  $k = (k_1, k_2)$  with  $k_\tau : \times_{n \in \mathcal{N}} k_\tau(b|n)$  and  $k_\tau(b|n) = (k_\tau^1(b|n), \dots, k_\tau^{n_\tau}(b|n))$ , and the transfer schedule  $y = (y_1, y_2)$  from the seller to the participants with  $y_\tau : \times_{n \in \mathcal{N}} y_\tau(n)$  for  $\tau = \{1, 2\}$ . Given  $\{\xi, k, y\}$ , each of  $N_1 + N_2$  potential bidders simultaneously makes their entry decision by assigning a probability  $p_\tau^i$  on their entry. Those who actually participate in the auction observe  $n$ , incur a participation cost  $c_\tau$ , obtain a monetary transfer  $y_\tau(n)$  and bid following a Nash bidding strategy  $\beta_\tau^i(\sigma|n)$ .

Now, let  $\pi_\tau^i(n|\xi, k)$  denote the expected payoff of a bidder in group  $\tau$  from the auction prior to drawing her private information. Then, the net gain of the bidder from participating in the auction,  $u_\tau^i(n|\cdot)$ , is given by

$$u_\tau^i(n|\xi, k, y) = \pi_\tau^i(n|\xi, k) + y_\tau(n) - c_\tau. \quad (4.1)$$

Throughout the paper we focus on the class of transfer schemes  $Y$  such that  $y_\tau(n)$  is decreasing in  $n$ . We also suppose that  $\pi_\tau(n|x_\tau)$  is decreasing in  $n$  so that  $u_\tau(n|x_\tau, y_\tau)$  is decreasing in  $n$ . Therefore, there exist a set of pairs of numbers  $n^* = (n_1^*, n_2^*)$  such that  $u_\tau(n_\tau^*, n_{-\tau}^*|\cdot) \geq 0 > u_\tau(n_\tau^* + 1, n_{-\tau}^*|\cdot)$  for some  $\tau = \{1, 2\}$ .<sup>52</sup> To keep the

<sup>52</sup>Throughout, we assume that, for any function  $\eta$ , the first and second arguments for  $n_1$  and  $n_2$  are exchangeable, *i.e.*,  $\eta(n_1, n_2|\cdot) \equiv \eta(n_2, n_1|\cdot)$

model general, we do not assume that the participation costs  $c$  must be moderate. Therefore, for some  $n_{-\tau}^*$ , we could have  $n_{\tau}^* = 0$  or  $n_{\tau}^* > N_{\tau}$ .

Hereafter, our analysis will proceed backward, beginning with the analysis of the second stage auction game. After we obtain the equilibrium bidding strategy and the associated *ex ante* expected payoffs from the auction, we will then investigate the entry decision in the first stage.

### 4.3 The Nash bidding strategy in the asymmetric auctions

By the Revelation Principle, the asymmetric auction analyzed here can be described as the incentive compatible (IC) direct selling mechanism. Let  $\sigma_{\tau} = (\sigma_{\tau}^1, \dots, \sigma_{\tau}^{n_{\tau}})$ . Then, given a report profile on signals  $\sigma = (\sigma_{\tau}, \sigma_{-\tau})$ , a direct mechanism is characterized as an allocation rule  $x(\sigma|n) = \{x_{\tau}^i(\sigma|n)\}_{i \in n_{\tau}, \tau \in \{1,2\}}$  and a payment rule  $\lambda(\sigma) = \{\lambda_{\tau}^i(\sigma)\}_{i \in n_{\tau}, \tau \in \{1,2\}}$ , where  $x_{\tau}^i(t) \in [0, 1]$  is the probability with which the  $i$ th bidder in group  $\tau$  obtains the item, and  $\lambda_{\tau}^i(t)$  is the expected payment the bidder makes to the auctioneer when  $\sigma$  is reported. In our analysis, we focus on the class of the mechanism such that the assignment rule  $x$  and the payment  $\lambda$  for bidders in the same group are identical, respectively. That is, we have  $x_{\tau}^i \equiv x_{\tau}$  and  $\lambda_{\tau}^i \equiv \lambda_{\tau}$  hold for all  $i$ . Hence, without using the superscript  $i$ , let  $w_{\tau}$  be the conditional expected payoff a group  $\tau$  bidder provided that her signal equals  $\sigma_{\tau}^i$ . If the remaining  $n - 1$  participants report  $(\sigma_{\tau}^{-i}, \sigma_{-\tau})$  truthfully, then the envelope integral formula makes  $w$  satisfy

$$w_{\tau}(\sigma_{\tau}^i|n, x_{\tau}) = w_{\tau}(0|n, x_{\tau}) + \int_0^{\sigma_{\tau}^i} \frac{d}{d\hat{\sigma}} v_{\tau}(\hat{\sigma}) x_{\tau}(\hat{\sigma}|n) d\hat{\sigma}.$$

Next, we derive the bidder's *ex ante* expected payoff from the asymmetric auction prior to drawing her signal  $\sigma_\tau^i$ . Since  $\sigma$  is uniformly distributed between 0 and 1, the expected payoff  $\pi_\tau(n|x_\tau) \equiv E[w_\tau(\sigma|\cdot)]$  is given by

$$\begin{aligned}\pi_\tau(n|x_\tau) &= \int_0^1 \int_0^{\sigma_\tau^i} \frac{d}{d\hat{\sigma}} v_\tau(\hat{\sigma}) x_\tau(\hat{\sigma}|n) d\hat{\sigma} d\sigma \\ &= \int_0^1 (1-\hat{\sigma}) \frac{d}{d\hat{\sigma}} v_\tau(\hat{\sigma}) x_\tau(\hat{\sigma}|n) d\hat{\sigma},\end{aligned}\quad (4.2)$$

where we normalize  $w_\tau(0|n, x_\tau) = 0$ . If there is a unique Nash bidding strategy denoted by  $\beta(\sigma|n)$ , then we have  $x(\sigma|n) \equiv \xi(\beta(\sigma))$  for all  $\sigma \in [0, 1]^n$ . Thus  $\pi_\tau(n|\xi_\tau) \equiv \pi_\tau(n|x_\tau)$  holds.

#### 4.4 Type-symmetric entry equilibria

A type symmetric equilibrium is an equilibrium in which all the bidders in the same group assign an identical probability on their participation. Suppose bidders in group  $\tau$  except  $i$  enter the auction with probability  $p_\tau$  and each bidder in group  $-\tau$  enters the auction with probability  $p_{-\tau}$ . In general, if  $N_t - k$  potential bidders in group  $t$  enter the auction with probability  $p_t$  and  $N_{-t} - \ell$  potential bidders in group  $-t$  enter with probability  $p_{-t}$ , then the probability that the number of actual entrants is equal to  $\hat{n} = (\hat{n}_t, \hat{n}_{-t})$  is given by

$$P_{\hat{n}, p}^{N_t - k, N_{-t} - \ell} \equiv \binom{N_t - k}{\hat{n}_t} \binom{N_{-t} - \ell}{\hat{n}_{-t}} [p_t]^{\hat{n}_t} [1 - p_t]^{N_t - k - \hat{n}_t} [p_{-t}]^{\hat{n}_{-t}} [1 - p_{-t}]^{N_{-t} - \ell - \hat{n}_{-t}}.$$

Therefore, provided that all the remaining potential bidders follow  $p \in (p_1, p_2)$ , the conditional expected gain of the  $i$ th bidder in group  $\tau$  from participating in the auction  $U_\tau^i(p_1, p_2|x_\tau, y_\tau)$  is written as

$$\begin{cases} U_1(p_1, p_2|x_1, y_1) \equiv \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2} P_{\hat{n},p}^{N_1-1, N_2} u_1(\hat{n}_1 + 1, \hat{n}_2|x_1, y_1) \\ U_2(p_1, p_2|x_2, y_2) \equiv \sum_{\hat{n}_1=0}^{N_1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n},p}^{N_1, N_2-1} u_2(\hat{n}_1, \hat{n}_2 + 1|x_2, y_2), \end{cases} \quad (4.3)$$

where we omit  $N$  and  $c = (c_1, c_2)$  on  $U_\tau^i$  since these are exogenous throughout our research.

Given  $\{x, y\}$ , the  $i$ th bidder in group  $\tau$  will randomize its participation if and only if  $U_\tau(p_1, p_2|x_\tau, y_\tau) = 0$ ; otherwise, he will choose to enter or stay out as a pure strategy. To clarify, let  $h_\tau : [0, 1]^{N_\tau-1} \times [0, 1]^{N-\tau} \rightarrow [0, 1]$  be the best response entry decision of the  $i$ th potential bidder in group  $\tau$ . Then this best response function can be described as

$$h_\tau(p_1, p_2|\cdot) \begin{cases} = 1 & \text{if } \{p_1, p_2|U_\tau(p_1, p_2|\cdot) > 0\} \\ = 0 & \text{if } \{p_1, p_2|U_\tau(p_1, p_2|\cdot) < 0\} \\ \in [0, 1] & \text{if } \{p_1, p_2|U_\tau(p_1, p_2|\cdot) = 0\}. \end{cases}$$

A type-symmetric mixed strategy entry equilibrium is characterized by a pair of probabilities  $(p_1^*, p_2^*)$ . Hence, the best response participation decision of the  $i$ th bidder in group  $\tau$ ,  $h_\tau$ , must be equal to  $p_\tau$  in equilibrium, implying  $(p_1^*, p_2^*)$  satisfies

$$h_\tau(p_1^*, p_2^*|\cdot) = p_\tau^*, \quad (4.4)$$

for all  $\tau \in \{1, 2\}$ . In other words, if we define  $A_\tau(p_1, p_2|\cdot) \equiv h_\tau(p_1, p_2|\cdot) - p_\tau$ , then  $p = (p_1, p_2)$  is a type symmetric equilibrium if and only if  $A_\tau(p|\cdot) = 0$  for all  $\tau = \{1, 2\}$ . The following proposition verifies the existence of such mixed-strategy equilibria in the asymmetric entry game with using this formulation.

**Proposition 9.** *There exists at least one mixed-strategy type-symmetric entry equilibrium in the participation game.*

The proof given in appendix is absolutely in line with a regular proof of the existence of a mixed-strategy Nash equilibrium.

#### 4.5 The property of the entry equilibria

The following figures depict  $A_\tau$ , as well as the mixed strategy entry equilibria as the intersections of  $A_\tau$  on  $(p_1, p_2)$  space.

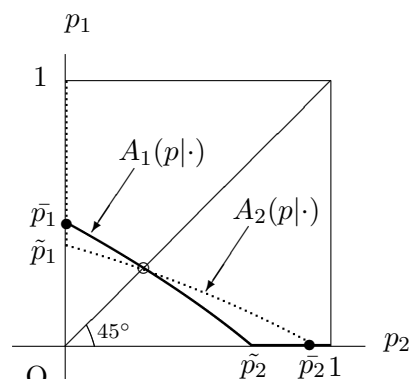


Figure (a)

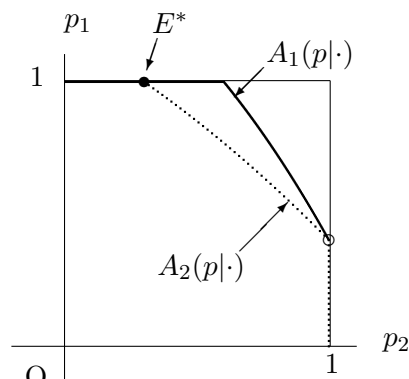


Figure (b)

We define  $\tilde{p}_\tau$  such that  $U_2(\tilde{p}_1, 0|x_2, y_2) = 0$  and  $U_1(0, \tilde{p}_2|x_1, y_1) = 0$ . Also, we define  $\bar{p}_\tau$  such that  $U_1(\bar{p}_1, 0|x_1, y_1) = 0$  and  $U_2(0, \bar{p}_2|x_2, y_2) = 0$ .

There are two types of mixed-strategy equilibria. To clarify, let  $G_\tau(p|x_\tau, y_\tau) \equiv \partial p_1 / \partial p_2|_{A_\tau=0}$  denote the absolute value of the slope of (A-1) on the  $p_1$ - $p_2$  square. On

taking total derivative of  $A_\tau(\cdot)$  with respect  $p_1$  and  $p_2$ , one obtains  $dA_\tau(p|x_\tau, y_\tau) = (\frac{\partial A_\tau}{\partial p_1})dp_1 + (\frac{\partial A_\tau}{\partial p_2})dp_2$ . So we have <sup>53</sup>

$$G_\tau(p|x_\tau, y_\tau) = \frac{\partial A_\tau / \partial p_2}{\partial A_\tau / \partial p_1} \quad (4.5)$$

if  $p_\tau \in (0, 1)$ . Let an “*odd*” *mixed-strategy type-symmetric equilibrium* be the equilibrium such that  $G_1(p^*|x, y) - G_2(p^*|x, y) \geq 0$ , and let an “*even*” *mixed-strategy type-symmetric equilibrium* be the equilibrium such that  $G_1(p^*|x, y) - G_2(p^*|x, y) < 0$ .

Since  $A_\tau(\cdot)$  is continuous and non-increasing, the number of *even* equilibria is always one less than the number of *odd* equilibria. Hence, by Proposition 9, there exists at least one *odd* equilibrium and an *even* equilibrium exists if and only if there are multiple equilibria. Furthermore, if all potential bidders are identical and  $y = \mathbf{0}$ , the symmetric equilibrium is always *even* since  $G_1(\rho, \rho) = N_2 / (N_1 - 1)$  and  $G_2(\rho, \rho) = (N_2 - 1) / N_1$  for any  $\rho \in (0, 1)$ .

To seek strategic interaction in the participation equilibrium, it is convenient to formulate the relative strength between the two groups by the difference in their expected gain from participation as follows:

<sup>53</sup>We can compute  $G$  by

$$\begin{aligned} \partial A_1 / \partial p_2 &= N_2 \cdot \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n}, p}^{N_1-1, N_2-1} [u_1(\hat{n}_1+1, \hat{n}_2+1|\cdot) - u_1(\hat{n}_1+1, \hat{n}_2|\cdot)], \\ \partial A_1 / \partial p_1 &= (N_1-1) \cdot \sum_{\hat{n}_1=0}^{N_1-2} \sum_{\hat{n}_2=0}^{N_2} P_{\hat{n}, p}^{N_1-2, N_2} [u_1(\hat{n}_1+2, \hat{n}_2|\cdot) - u_1(\hat{n}_1+1, \hat{n}_2|\cdot)], \\ \partial A_2 / \partial p_2 &= (N_2-1) \cdot \sum_{\hat{n}_1=0}^{N_1} \sum_{\hat{n}_2=0}^{N_2-2} P_{\hat{n}, p}^{N_1, N_2-2} [u_2(\hat{n}_1, \hat{n}_2+2|\cdot) - u_2(\hat{n}_1, \hat{n}_2+1|\cdot)], \\ \partial A_2 / \partial p_1 &= N_1 \cdot \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n}, p}^{N_1-1, N_2-1} [u_2(\hat{n}_1+1, \hat{n}_2+1|\cdot) - u_2(\hat{n}_1+1, \hat{n}_2|\cdot)]. \end{aligned}$$

**Definition 1.** *The group 1 potential bidders are **more profitable** than the group 2 potential bidders if and only if i)  $u_1(n_1 + 1, n_2|x_1, y_1) \geq u_2(n_1, n_2 + 1|x_2, y_2)$  holds for some  $x$  and  $y$  and ii)  $u_1(n_1, n_1|x_1, y_1) \geq u_2(n_1, n_2|x_2, y_2)$  holds for such  $x$  and  $y$ .*

Condition i) implies that the *ex ante* payoff of a particular bidder is monotonically increased by the change of her type from 2 to 1, whereas ii) involves that the *ex ante* payoff of a type 1 potential bidder is always greater than that of a type 2 bidder in the auction. If the mechanisms are *ex post* efficient, a more profitable bidder is equivalent to the strong bidder in the sense that her value distribution stochastically dominates the value distribution of a weak bidder.<sup>54</sup> Then, one obtains the following lemma about equilibrium participation decision.

**Proposition 10.** *Suppose a potential bidder in one group is **more profitable** than a potential bidder in the other group for some mechanism and transfer scheme. Then, the probability with which a less profitable bidder enters the auction is greater than the probability with which a more profitable bidder enters the auction if each potential bidder is indifferent between participating and staying out in equilibrium.*

See appendix for proof. A symmetric equilibrium with *ex ante* identical bidders corresponds to the special case where the equations in condition i) and ii) hold

<sup>54</sup>If a mechanism is *ex post* efficient, we have

$$\begin{aligned}\pi_1(n_1 + 1, n_2|x) &= \int_v (1 - F_1(v))(F_1(v))^{n_1}(F_2(v))^{n_2} dv, \\ \pi_2(n_1, n_2 + 1|x) &= \int_v (1 - F_2(v))(F_1(v))^{n_1}(F_2(v))^{n_2} dv,\end{aligned}$$

for any  $n$ . Hence,  $F_1(v) \leq F_2(v)$  for any  $v$  implies  $\pi_1(n_1 + 1, n_2|x) \geq \pi_2(n_1, n_2 + 1|x)$ . Also, under the *ex post* efficient mechanism, we have

$$\begin{aligned}\pi_1(n_1, n_2|x) &= \int_v (1 - F_1(v))(F_1(v))^{n_1-1}(F_2(v))^{n_2} dv \\ &= \int_v [F_2(v) - F_1(v)F_2(v)](F_1(v))^{n_1-1}(F_2(v))^{n_2-1} dv \\ \pi_2(n_1, n_2|x) &= \int_v (1 - F_2(v))(F_1(v))^{n_1}(F_2(v))^{n_2-1} dv \\ &= \int_v [F_1(v) - F_1(v)F_2(v)](F_1(v))^{n_1-1}(F_2(v))^{n_2-1} dv\end{aligned}$$

for any  $n$ . Thus, if  $F_1(v) \leq F_2(v)$  for any  $v$ , then  $\pi_1(n_1, n_2|x) \geq \pi_2(n_1, n_2|x)$ .



with equality, as is drawn in figure (a). On the other hand, if asymmetry between two groups is sufficiently large, no *even* equilibrium is likely. The marginal case is shown in the figure (b), in which an *even* equilibrium will disappear if group 1 bidders become more profitable.<sup>55</sup>

If potential bidders across groups are identical and  $c$  is moderate, there exist other two asymmetric equilibria. The asymmetric equilibrium analyzed by McAfee and McMillan (1987b), Engelbrecht-Wiggans (1993) is the case where  $p_\tau^* = 1$  and  $p_{-\tau}^* = 0$  for some  $\tau$ . Depending on  $N_1$  and  $N_2$ , there could be many other asymmetric equilibria *e.g.*,  $p_1^* = 1$  and  $p_2^* \in (0, 1)$ , as shown in figure (b), where group 1 bidders obtain positive expected rents.

The auctioneer has no reason to keep the bidders obtaining strictly positive rents. The following lemma shows that full extraction of rents is trivially possible by a entry fee.

**Lemma 3.** *For any  $p$ , there exists a constant participation fee schedule  $y_{p,x,y}^0 = (y_{1,p,x_1,y_1}^0, y_{2,p,x_2,y_2}^0) \in \mathfrak{R}^2$  such that full extraction of rents is possible for some  $x \in X$  and  $y \in Y$ . Moreover,  $y_{p,x,y}^0$  implements the rent extraction holding  $p$  constant.*

<sup>55</sup>Sufficiency for multiple equilibria is  $\bar{p}_\tau \geq \tilde{p}_\tau$  for all  $\tau$ . Intuitively, this is the case where, if each party commits to assigning its maximum probability  $\bar{p}_\tau$ , then there is no room for the other group to participate profitably.

*Proof.* Set  $y_{\tau,p,x_{\tau},y_{\tau}}^0$  such that  $0 = y_{\tau,p,x_{\tau},y_{\tau}}^0 + U_{\tau}(p|x_{\tau}, y_{\tau})$  for some  $p, x$  and  $y$ . Then, by (4.1) and (4.3),

$$\begin{aligned} 0 &= y_{\tau,p,x_{\tau},y_{\tau}}^0 + \sum_{\hat{n}_{\tau}=0}^{N_{\tau}-1} \sum_{\hat{n}_{-\tau}=0}^{N_{-\tau}} P_{\hat{n},p}^{N_{\tau}-1,N_{-\tau}} [\pi_{\tau}(\hat{n}_{\tau} + 1, \hat{n}_{-\tau}|x_{\tau}) + y_{\tau} - c_{\tau}] \\ &= \sum_{\hat{n}_{\tau}=0}^{N_{\tau}-1} \sum_{\hat{n}_{-\tau}=0}^{N_{-\tau}} P_{\hat{n},p}^{N_{\tau}-1,N_{-\tau}} [\pi_{\tau}(\hat{n}_{\tau} + 1, \hat{n}_{-\tau}|x_{\tau}) + y_{\tau} + y_{\tau,p,x_{\tau},y_{\tau}}^0 - c_{\tau}] \\ &= U_{\tau}(p|x_{\tau}, y_{\tau} + y_{\tau,p,x_{\tau},y_{\tau}}^0), \end{aligned}$$

for any  $p$  and  $x$ .  $x$  and  $p$  are unchanged throughout.  $\square$

A simple entry fee schedule, although it might be discriminatory, allows the seller to extract full rents without disturbing equilibrium  $p$ . Therefore, the optimal design problem in auctions with asymmetric potential bidders is equivalent to the maximization problem of the social surplus.

The social surplus associated with the transaction here is defined as the winning bidders valuation for the item minus the sum of participation costs incurred by participants. Let  $\beta(n|\cdot)$  be the incentive compatible expected payment from the winning bidder to the seller in the auction taking  $n$  as given. Then the auction revenue  $R(p|\cdot)$  is written as

$$R(p|x, y) = \sum_{\hat{n}_1=0}^{N_1} \sum_{\hat{n}_2=0}^{N_2} P_{\hat{n},p}^N [\beta(\hat{n}_1, \hat{n}_2|x) - y(\hat{n}_1, \hat{n}_2)].$$

From (4.1), the sum of bidder's *ex ante* expected payoffs  $U(p|x, y) = N_1 p_1 U_1(p|x_1, y_1) + N_2 p_2 U_2(p|x_2, y_2)$  is

$$U(p|x, y) = \sum_{\hat{n}_1=0}^{N_1} \sum_{\hat{n}_2=0}^{N_2} P_{\hat{n},p}^N [V(\hat{n}_1, \hat{n}_2|x) - \beta(\hat{n}_1, \hat{n}_2|x) + y(\hat{n}_1, \hat{n}_2)] - N_1 p_1 c_1 - N_2 p_2 c_2,$$

where  $V(\cdot)$  is the expected valuation of the winning bidder given  $\hat{n}_1, \hat{n}_2$  and  $x$ . On the other hand, the total surplus  $S(\cdot)$  is given by

$$\begin{aligned} S(p|x) &= R(p|x, y) + U(p|x, y) \\ &= \sum_{\hat{n}_1=0}^{N_1} \sum_{\hat{n}_2=0}^{N_2} P_{\hat{n}_1, p}^{N_1} V(\hat{n}_1, \hat{n}_2|x) - N_1 p_1 c_1 - N_2 p_2 c_2. \end{aligned} \quad (4.6)$$

By lemma 3,  $y_{\tau, p, x, y}^0$  enables the auctioneer to extract bidders' rents at all for any  $p$ , *i.e.*,  $U(p|x, y_{p, x, y}^0) = 0$ . Hence, one obtains

$$S(p|x) = R(p|x, y_{p, x, y}^0).$$

It implies that, if the seller sets a monetary transfer scheme such that bidder's rents will be zero, then the seller revenue is identical to the social surplus for any mechanisms.

Also, (4.6) may remind us the revenue equivalence theorem for asymmetric bidders with entry, as described in the following statement:

**Theorem 1.** *Suppose bidders' rents are fully extracted. If any two mechanisms have the same probability assignment functions and induce equal entry, then the two mechanisms generate the same revenue for the seller.*

This insists that the pure transfer  $y$  is redundant on the expected revenue as long as rents are fully extracted and the equilibrium entry  $p$  is unchanged. It follows that auctioneers have many alternative transfer schedule  $y$  that do not influence  $p$ .

In the rest of this section, we explore the maximization problem over  $S(p|x)$  to find the upper bound of social surplus  $\hat{S}$ . Among the arguments on  $S$ , we first

investigate  $x$ , then control  $p$  to seek  $\hat{S}$ . It is trivially true that any *ex post* inefficiency stemming from distortive allocation or a positive reservation price decreases the social surplus.

**Proposition 11.** *Let  $x^*$  represent the ex post efficient mechanisms. For any  $p \in [0, 1] \times [0, 1]$ , the social surplus is maximized if and only if the mechanism is ex post efficient, namely for any  $p$   $S(p|x^*) > S(p|x) \quad \forall x \in X \setminus x^*$ .*

*Proof.*  $V(n|x^*) \geq V(n|x)$  holds for any  $x \in X$ . Hence, by (4.6),  $S(p|x^*) \geq S(p|x)$  holds for any  $p$ . □

Once focusing on the *ex post* efficient mechanisms, we can discuss a useful theorem for equilibrium analysis shown as follows. This theorem provides a relationship between the *ex ante* payoff for each potential bidder and the expected marginal contribution to the social surplus of the bidder.

**Theorem 2.** *Let  $v_\tau^{(1)}$  be the highest valuation among group  $\tau$  bidders. Let  $\phi_\tau(\cdot|x)$  be a matching function such that the bidder with  $v_\tau^{(1)}$  and the bidder with  $v_{-\tau}^{(1)}$  tie under some mechanism  $x$  if and only if  $v_{-\tau}^{(1)} = \phi_\tau(v_\tau^{(1)}|x)$ . Suppose  $\phi_\tau(v_\tau) \geq v_\tau$  for some  $\tau$ . Then, for any  $n_\tau$*

$$\begin{cases} \pi_1(n_1+1, n_2|x) = V(n_1+1, n_2|x) - V(n_1, n_2|x), \\ \pi_2(n_1, n_2+1|x) = V(n_1, n_2+1|x) - V(n_1, n_2|x), \end{cases} \quad (4.7)$$

*if and only if  $\phi_\tau(v_\tau) = v_\tau$ .*

See appendix for proof. This property is first introduced by Engelbrecht-Wiggans (1993) in IPV setting. We extend the results to the asymmetric private value environment and gives a sufficient condition for this property to hold. Most of the

standard auctions satisfy the condition  $\phi_\tau(v_\tau) \geq v_\tau$ . For example, in asymmetric first price auctions with a strong bidder and a weak bidder, we have  $\phi_\tau(v_\tau) > v_\tau$ , implying (4.7) does not hold.<sup>56</sup> On the other hand, second price auctions with a positive and constant reservation price satisfies  $\phi_\tau(v_\tau) = v_\tau$ . These assure that *ex post* efficiency in allocation is not necessary for the marginal contribution theorem to hold as shown in Engelbrecht-Wiggans (1993). Obviously, the amount of contribution by one more additional bidder under the *ex post* inefficient mechanism is strictly smaller since  $V(n_\tau+1, n_{-\tau}|x^*) - V(n_\tau, n_{-\tau}|x^*) > V(n_\tau+1, n_{-\tau}|x) - V(n_\tau, n_{-\tau}|x)$  for any  $x \in X \setminus \{x^*\}$ .

Now we return to the maximization problem of  $S$ . Taking partial derivative of  $S$  characterized in (4.6) with respect to  $p$  gives the first order condition as follows

$$\begin{cases} \frac{\partial S}{\partial p_1} = N_1 \cdot \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2} P_{\hat{n},p}^{N_1-1, N_2} [V(n_1+1, n_2|x) - V(n_1, n_2|x)] - N_1 c_1 \\ \frac{\partial S}{\partial p_2} = N_2 \cdot \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2-1} P_{\hat{n},p}^{N_1, N_2-1} [V(n_1, n_2+1|x) - V(n_1, n_2|x)] - N_2 c_2. \end{cases} \quad (4.8)$$

<sup>56</sup>Suppose there are two bidders  $j = \{1, 2\}$  whose value distribution is  $F_j(v)$  on  $[\bar{v}, \underline{v}]$  and  $f_j(v)$  denotes the corresponding density. Assume  $\ln F_j(v)$  is supermodular. Let  $\beta_j$  be the equilibrium bidding functions. The range of the equilibrium bidding functions should be identical, so that  $\beta_1(\bar{v}) = \beta_2(\bar{v})$  holds.

Note that the second derivative is negative under the *ex post* efficient mechanisms.<sup>57</sup>

Then, by Theorem 2, one can rewrite (4.8) as

$$\begin{cases} \frac{\partial S}{\partial p_1} = N_1 \cdot \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2} P_{\hat{n},p}^{N_1-1, N_2} [\pi_1(n_1+1, n_2|x) - c_1] \\ \frac{\partial S}{\partial p_2} = N_2 \cdot \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2-1} P_{\hat{n},p}^{N_1, N_2-1} [\pi_2(n_1, n_2+1|x) - c_2]. \end{cases} \quad (4.9)$$

Recall that (4.9) coincides with  $A_\tau(\cdot|x^*, 0)$ . Therefore, (4.9) vanishes if  $(p_1^*, p_2^*) \in (0, 1) \times (0, 1)$  and  $\partial S(p|x^*, \cdot)/\partial p_\tau \leq 0$  or  $\partial S(p|x^*, \cdot)/\partial p_\tau \geq 0$  if  $p_\tau^* = 0$  or 1. These imply that the social gain from an incremental change in  $p_\tau$  equals the social cost of an incremental change in the probability for each  $\tau$ .<sup>58</sup> We thus establish the following proposition about efficient entry.

**Proposition 12.** *Suppose a mechanism is ex post efficient and involves no transfer. Then, the bidder's profit maximizing entry decision is the necessary and sufficient condition for efficient entry in the sense that the social gain from entry equals the cost of the entry.*

The result is quite reasonable, as we already know that the bidder's expected payoff is identical to the marginal contribution to the social surplus under  $x^*$ . Since the agent's problem is the same as the society's problem, the bidder's rational participation decision leads to efficient outcome in entry.

<sup>57</sup>Show  $\partial^2 S(p|x^*, \cdot)/\partial p_1^2 < 0$ . Under the *ex post* efficient mechanism,  $V(n_1+2, n_2|x^*, \cdot) - 2V(n_1+1, n_2|x^*, \cdot) + V(n_1, n_2|x^*, \cdot) = \pi_1(n_1+1, n_2|x^*, \cdot) - \pi_1(n_1, n_2|x^*, \cdot)$  holds. Furthermore, one obtains  $\pi_1(n_1+1, n_2|x^*, \cdot) = \int_0^1 (1-\sigma) \frac{d}{d\sigma} v^s(\sigma) \sigma^{n_1} \cdot (\phi_1(\sigma))^{n_2} d\sigma$ , where  $\phi_\tau(\cdot) \equiv v_\tau^{-1}(v_\tau(\sigma))$  is a matching function. Then, for any  $n_1$  and  $n_2$ ,  $\pi_1(n_1+1, n_2|\cdot) - \pi_1(n_1, n_2|x^*, \cdot) = - \int_0^1 (1-\sigma)^2 \frac{d}{d\sigma} v_1(\sigma) \sigma^{n_1-1} \cdot (\phi_1(\sigma))^{n_2} d\sigma < 0$ . The same is true for  $\partial^2 S(p_1, p_2|x^*, \cdot)/\partial p_2^2 < 0$ .

<sup>58</sup>Also, if the gain is greater (or less) than the costs, deterministic participation (or staying out) occurs.

The sufficient condition for our maximization problem is a little more involving than in the symmetric case. The following lemma demonstrates that, as a result of Hessian analysis over  $S(p|x)$ , any local maximum (or saddle) point on  $S(p|x)$  is formed at the intersection between  $A_r(\cdot)$  if the mechanism is *ex post* efficient without monetary transfer.

**Lemma 4.** *If  $x = x^*$  and  $y = \mathbf{0}$ , then odd equilibria form a local maximum and even equilibria form a saddle point on the social welfare function  $S(p|x^*)$ .*

See appendix for proof. Again, the linkage between  $A(\cdot)$  and  $S(p|x^*)$  is shown here under the *ex post* efficient mechanisms with no transfer. The proof demonstrates that the gradient of  $S(p|x^*)$  is determined by the positional relationship between  $A_1(\cdot)$  and  $A_2(\cdot)$ .

If an analysis focuses on a symmetric equilibrium with *ex ante* symmetric bidders, efficiency and optimality in entry are equivalent. Taking into account multiple equilibria, however, efficient entry may not imply optimal entry. Different equilibria create different level of social surplus due to the difference in coordination costs associated with the randomness of the actual number of participants. Lemma 4 implies that *even* equilibria entail more waste of social welfare than *odd* equilibria, and hence lower revenue. For instance, the symmetric equilibrium in a case of symmetric potential bidders suffers the highest coordination costs among three equilibria.

There is at least one *odd* equilibrium in the asymmetric auctions with endogenous participation. Also, in case of multiple equilibria, one of the *odd* equilibria yields the highest social welfare. Hence,  $\hat{S}$  is achieved at an odd equilibrium under the *ex post* efficient mechanisms. The following proposition addresses that this is only in the case of *ex post* efficient mechanisms.

**Proposition 13.** *The social surplus created in an odd equilibrium attains  $\hat{S}$  if the mechanism is ex post efficient and no transfer is used. Efficient entry is necessary for socially optimal entry, but not sufficient.*

*Proof.* Proof for “if” part is obvious by proposition 11. The proof for “only if” part is shown as follows. Let  $\hat{p}^x$  denote the maximizer for  $S(p|x)$  for any  $x$ . By construction,  $S(\hat{p}^{x^*}|x^*) \geq S(p|x^*)$  for any  $p$ . By proposition 11,  $S(\hat{p}^x|x^*) > S(\hat{p}^x|x)$  for any  $x \neq x^*$ . Hence,  $S(\hat{p}^{x^*}|x^*) > S(\hat{p}^x|x)$  for any  $x \neq x^*$ .  $\square$

We conclude this section with considering implementation in the entry game with multiple equilibria. The auctioneer has two devices to influence entry *i.e.*,  $x$  and  $y$ . For example, setting a discriminatory reservation price which favors a group of bidders affects  $A_\tau(\cdot)$ , and hence the equilibrium formulation. Also, bidding credits may help a particular group of potential bidders to participate more frequently and discourage the remaining bidders’ participation. Furthermore, monetary transfer  $y$  enables the seller to control  $A_\tau(\cdot)$ , or extract more surplus from a group of bidders. Should the auctioneer choose any  $x \neq x^*$  to reach the best equilibrium?

The following lemma illustrates that the seller can implement any  $p$  as a unique equilibrium by controlling  $y \in Y$ .<sup>59</sup>

**Lemma 5.** *Given  $x \in X$ , there exists a set of transfer schedule  $\hat{Y}_{p,x,y}$  with which any  $p$  will be induced as a unique mixed-strategy entry equilibrium.*

See appendix for proof. Recall Lemma 3 insisting that full rent extraction is possible at any  $p$  by any  $y_{p,x,y}^0 \in Y_{p,x,y}^0$ . Furthermore, we already know that  $\hat{S}$  is

<sup>59</sup>It is primarily because  $y$  is a function of  $n_1$  and  $n_2$ , rather than a negative constant variable *i.e.*, a fixed entry fee. However, nothing would be gained in the symmetric model if  $y$  is a function of  $n$ .



never achieved unless the mechanism is *ex post* efficient in proposition 13. Hence, the optimal mechanisms in the auctions with asymmetric potential bidders involves a participation control as mentioned in the following proposition.

**Proposition 14.** *The participation game has a unique mixed-strategy type-symmetric equilibrium in which the auctioneer's revenue is maximized, if and only if the auctioneer employs the ex post efficient mechanism with an appropriately-chosen transfer scheme.*

*Proof.* By lemma 5, there exists a set of  $\hat{y}$  which induces  $\hat{p}$  as a unique odd equilibrium. Furthermore, if type  $\tau$  potential bidders have positive expected rents, the auctioneer can extract them at all by setting  $\hat{y}_\tau^0 = \hat{y}_\tau + y_\tau^0$ . Let  $\hat{y}^0 = (\hat{y}_\tau^0, \hat{y}_{-\tau}^0)$ . Then, by Lemma 5,  $\hat{y}$  and  $\hat{y}^0$  induce the same entry if and only if it is unique.  $\square$

#### 4.5.1 Affiliated private value

Suppose there exists  $b_\tau(v|x)$  for some  $x$  such that the bidder who has the highest  $b$  wins the item. Define  $\phi_\tau(v|x) = b_{-\tau}^{-1}(b_\tau(v|x)|x)$  and  $F_\tau(v|x) = F_\tau(\phi_{-\tau}(v|x))$ . Then, (4.2) becomes

$$\begin{aligned}\pi_1(n_1, n_2|x) &= \int_z \int_v (1 - F_1(v|z))(F_1(v|z))^{n_1-1}(F_2(v|z))^{n_2} dv g(z) dz, \\ &= \int_z \int_v v[(F_1(v|z))^{n_1}(F_2(v|z))^{n_2}]' dv dz - \int_z \int_v v[(F_1(v|z))^{n_1-1}(F_2(v|z))^{n_2}]' dv g(z) dz \\ &= V(n_1, n_2|x^*) - V(n_1 - 1, n_2|x^*),\end{aligned}$$

which extends the theorem 2 to the affiliated private value paradigm. Now we have extended the results to the asymmetric APV environment.

## 4.6 Discussion

We show that, if endogenous participation is accounted for, *ex post* efficiency is necessary for optimal mechanism. The results contradict the existing theorems for optimal design with a fixed set of asymmetric bidders, which insist that the *ex post* efficient mechanisms are not optimal (See Myerson (1981), McAfee and McMillan (1989), and Bulow and Roberts (1989)). Our model illustrates that costs of introducing a distortive assignment rule for rent extraction always outweigh the benefits due to a serious inefficiency in participation.

Proposition 14 also impacts on the ranking theorems with IPV asymmetric auctions. Vickrey (1961) showed that there is no general ranking between first and second price asymmetric auctions, if the number of bidders is exogenously determined.<sup>60</sup> Considering endogenous participation, we obtain the clear ranking; Second price mechanisms always dominate first price mechanisms. In addition, Milgrom and Weber (1982) suggest that the second price mechanisms yields higher revenue than the first price ones if signals are affiliated. Therefore, our proposition about the superiority of second price mechanisms still holds in case of asymmetric APV environments.

The proposition that efficient entry is a necessary condition for optimal outcome is also referred to in Levin and Smith (1994).<sup>61</sup> They address that more potential bidders exceed the point of transition between pure and mixed entry strategies result

<sup>60</sup>The existence of equilibrium bidding function in the asymmetric first price auctions with fixed number of bidders is shown by Lebrun (1999). In general, weak bidders would bid more aggressively than strong bidders in the first price auction, resulting in the *ex post* inefficient allocation. See Maskin and Riley (2000).

<sup>61</sup>Ye (2004) also shows the advantage of asymmetric equilibrium by using Jensen's inequality. However, the results are crucially dependent upon the assumption that potential bidders are *ex ante* identical.

in the waist of social welfare.<sup>62</sup> Our approach is a non-trivial extension from theirs since we provide a generalized scheme which allows one to evaluate revenue across equilibria with asymmetric potential bidders.

For ways of promoting competition in the asymmetric auctions with participation, our analysis relates to Ayres and Cramton (1996) and Gilbert and Klemperer (2000). Motivated by the auctioneers' concern about insufficient competition among well-qualified bidders, they explore whether subsidizing weak buyers through a distortive allocation rule increases revenue. Both conclude that promoting entry by weak buyers will enhance revenue. Since an *even* equilibrium is a saddle point on  $S(\cdot|x^*)$ , the change of an allocation rule from  $x^*$  to some  $x'$  may induce another  $p$  which creates greater  $S$  despite some efficiency loss in allocation. However, our study suggests that the outcome is sub-optimal. The first best outcome is achieved only through an *ex post* efficient allocation with a pure transfer.

A transfer may often be seen in the real world procurement auction as the requirement for a higher financial guarantee for bidders. It is costly for bidders but beneficial to the auctioneer by reducing risk of facing default. The governments spending for improving SMEs' access can also be considered to be a transfer. An

<sup>62</sup>This argument is true without symmetry assumption if mechanisms are *ex post* efficient. Since  $S(p_1, p_2|x, N_1, N_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2} P_{\hat{n}, p}^{N_1-1, N_2} [p_1 \cdot V(n_1+1, n_2|x) + (1-p_1) \cdot V(n_1, n_2|x)] - N_1 p_1 c_1 - N_2 p_2 c_2 = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2-1} P_{\hat{n}, p}^{N_1, N_2-1} [p_2 \cdot V(n_1, n_2+1|x) + (1-p_2) \cdot V(n_1, n_2|x)] - N_1 p_1 c_1 - N_2 p_2 c_2$ ,  $S(p_1, p_2|x, N_1-1, N_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2} P_{\hat{n}, p}^{N_1, N_2-1} V(n_1, n_2|x) - (N_1-1)p_1 c_1 - N_2 p_2 c_2$  and  $S(p_1, p_2|x, N_1, N_2-1) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2-1} P_{\hat{n}, p}^{N_1, N_2-1} V(n_1, n_2|x) - N_1 p_1 c_1 - (N_2-1)p_2 c_2$  for for any  $p_1, p_2$  and  $x$ , we have

$$\begin{cases} S(p|x^*, N_1, N_2) - S(p|x^*, N_1-1, N_2) = p_1 \cdot \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2} P_{\hat{n}, p}^{N_1-1, N_2} [V(n_1+1, n_2|\cdot) - V(n_1, n_2|\cdot)] - p_1 \cdot c_1 \\ S(p|x^*, N_1, N_2) - S(p|x^*, N_1, N_2-1) = p_2 \cdot \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2-1} P_{\hat{n}, p}^{N_1, N_2-1} [V(n_1, n_2+1|\cdot) - V(n_1, n_2|\cdot)] - p_2 \cdot c_2. \end{cases}$$

If  $x = x^*$ , then, by Theorem 2, both are equal to zero. These indicate that if a potential bidder, regardless of its type, is eliminated and if the remaining potential bidders still use  $p$ , then social welfare is unchanged. If the remaining potential bidders choose  $p'$  according to their new best response accounting one less potential bidders, then, social surplus gets increased by construction.

important aspect on the participation control, those including such implicit one, is that any change in participation from the optimal equilibrium results in both efficiency loss due to coordination costs and fluctuation in participation.

## 4.7 Conclusion

Over the decade, the model of auctions with endogenous participation provides a striking result that both efficiency and revenue maximization can be achieved simultaneously. Despite the contribution of the endogenous participation models, they crucially depend on the assumption that the potential bidders are *ex ante* the same. Little progress has been made in the theory of auction with asymmetric endogenous entry.

The relaxation of symmetric assumption is not a trivial extension from the existing symmetric model. First, with symmetric bidders, optimal design problems boil down to the optimal choice of reservation price, as investigated in Riley and Samuelson (1981). Introducing asymmetry, the optimal design problem becomes more complicated. As discovered in Myerson (1981), the appropriately-chosen distortive mechanisms enhance revenue. This proposition, however, absolutely ignores the effects of potential bidder's participation. Accounting for the rational decision of potential bidders' participation, any rent extraction by distortive allocation causes inefficient entry, and hence, simple auctions are optimal. The results provide a new interpretation for the widespread use of simple auctions.

Second, it makes the endogenous entry model be applicable for the more general environment. In the procurement auctions for international projects, for example, contractors from around the world bid. Based on the fact that the traveling costs as

well as currency values differs from countries to countries, it is impossible to suppose that all the potential bidders are *ex ante* the same. Almost all empirical models for auctions with endogenous participation, so far, has been based on the symmetric assumption (*e.g.* Li and Zheng (2006)). I hope that our model contributes to enrichment of the empirical analysis for auctions with endogenous participation.

## CHAPTER 5

### CONCLUSION

This dissertation examines the impact of small businesses on bidder participation and contract prices in government procurement auctions. The evidence indicates that although they are hampered by relatively high production costs and limited knowledge when bidding, the presence of small businesses has non-trivial effects on government procurement activities.

Despite controversy over how much society is paying extra costs, set-asides are widely used in real world public procurement for the enhancement of small business access to government procurement. The analysis in chapter 2 is devoted to the empirical measurement of the effect of small business set-asides on government procurement costs, bidding behavior, and bidder participation in competitive bidding. Applying nonparametric estimation methodologies to bidding data from a large number of construction procurement auctions, this study finds that the program dramatically increases the small business participation but is almost neutral with respect to the procurement costs. Empirical analysis also suggests that the cost to the program set-aside auctions is exaggerated if one considers only the premium paid to small businesses for contracts allocated to them. I found that set-asides

increase the auctioneer's welfare, yielding more competitive pressure on advantaged bidders. The subsidized SMEs drive non-subsidized bidders to give up more of the gain on the contracts they award.

Chapter 3 provides a theoretical investigation of the influence of subcontract auctions on a primary contractor's bidding behavior in procurement auctions. Most of the auction literature implicitly assumes that the players are the agents who send a message directly to the principal (auctioneer). In procurement auctions, goods and services are typically produced by a team of firms (main and sub firms), where the lower-tiered subcontractors and suppliers are non-negligible players who also possess private information. The theoretical Bayesian game model in this research shows that the lower tiered producers and suppliers reveal and sacrifice more informational rent as competition in the downstream auction becomes more intense. Their aggressive bidding in the upstream auction, in turn, helps their prime contractor win in the downstream auctions. The main contribution of this research lies in formulating an auction model including the vertically related production system that can be seen in most of the industries. The theory suggests that an additional entrant to the downstream auction results in strengthening a given prime contractor's bargaining power against her subcontractors. It implies that prime contractors have their cost distribution endogenously determined by the number of bidders and the reservation price in the downstream auction. The application of the theory is widespread and can in particular be applied to joint bidding or bid consortia. Even though member firms have a close trust relationship with each other, each member owns private information, which creates the possibility for each member firm to obtain a rent against the consortium as a whole.

In the final essay, I construct a theoretical model of participation in auctions. Departing from the existing model of auctions with endogenous participation, which is relying on the assumption that the potential bidders are *ex ante* the same, I analyze the strategic interaction to participate in an auction between advantaged and disadvantaged bidders. In fact, little progress has been made in the theory of auction with asymmetric endogenous entry because of the complexity in the optimal design problem with asymmetric bidders. A remarkable evidence found in this research is that accounting for the rational decision of potential bidders' participation, any rent extraction by distortive allocation causes inefficient entry, and hence, simple auctions are optimal. The results provide a new interpretation for the widespread use of simple auctions even in the situation that some potential bidders are obviously more advantaged than others.

I believe that these three essays contribute to enrichment of the empirical and theoretical analysis for procurement auctions.



## APPENDIX

## APPENDIX FOR CHAPTER 2

### 2.A An alternative proof for Zhang and Guler (2005)

The inverse bidding function in asymmetric auction,  $\theta_i(b_i)$ , satisfies the following FOC.

$$\theta_i(b_i) = b_i - \frac{1}{\sum_{-i} \frac{f_{-i}(\theta_{-i}(b_i))\theta'_{-i}(b_i)}{1-F_{-i}(\theta_{-i}(b_i))}} \quad \forall i = 1, \dots, n.$$

If the inverse bidding function is monotone, a change-of-variables argument holds

$$\begin{aligned} G_{-i}(z) &= F_{-i}(\theta_{-i}(z)) \\ g_{-i}(z) &= f_{-i}(\theta_{-i}(z))\theta'_{-i}(z) \quad \forall z \in [\underline{b}, \bar{b}]. \end{aligned}$$

So IBFs can be simplified as

$$\theta_i(b_i) = b_i - \frac{1}{\sum_{-i} \frac{g_{-i}(b_i)}{1-G_{-i}(b_i)}} \quad \forall i = 1, \dots, n.$$

Without loss of generality, set  $i = 1$ . IBF for bidder 1 is rewritten as

$$\theta_1(b_1) = b_1 - \frac{1}{\left\{ \frac{g_2(b_1)}{1-G_2(b_1)} + \frac{g_3(b_1)}{1-G_3(b_1)} + \dots + \frac{g_n(b_1)}{1-G_n(b_1)} \right\}}. \quad (\text{A-1})$$

Reducing to a common denominator, the expression in the curly bracket is rewritten as

$$\frac{\left[ \prod_{k \neq \{1,2\}} \{1 - G_k(b_1)\} \right] g_2(b_1) + \dots + \left[ \prod_{k \neq \{1,n\}} \{1 - G_k(b_1)\} \right] g_n(b_1)}{\prod_{k \neq 1} \{1 - G_k(b_1)\}}.$$

Let  $1 - G_1^*(b_1) = \prod_{k \neq 1} \{1 - G_k(b_1)\}$ . Assume  $\exists g_1^*(b_1) = \frac{\partial G_1^*(b_1)}{\partial b_1}$ . Then

$$g_1^*(b_1) = \left[ \prod_{k \neq \{1,2\}} \{1 - G_k(b_1)\} \right] g_2(b_1) + \dots + \left[ \prod_{k \neq \{1,n\}} \{1 - G_k(b_1)\} \right] g_n(b_1).$$

So (A-1) can be rewritten as

$$c_1 = b_1 - \frac{1 - G_1^*(b_1)}{g_1^*(b_1)}.$$

Now we interpret the non-negative strictly increasing function " $G_1^*(\cdot)$ ". Let  $B_1 = \min_{k \neq 1} b_k$ . Then,

$$\begin{aligned} 1 - G_1^*(x) &\equiv \prod_{k \neq 1} \{1 - G_k(x)\} = \Pr(x \leq B_1) \\ g_1^*(x) &\equiv \frac{\partial G_1^*(x)}{\partial b_1} = \Pr(x = B_1) \end{aligned}$$

Suppose  $j = 1, \dots, m$  auctions with identical set of bidders. Let  $B_{1,j} = \min_{k \neq 1} b_{1,j}$  for all  $j$ . Then,

$$\begin{aligned} 1 - \hat{G}_1^*(B) &= \frac{1}{m} \sum_{j=1}^m \mathbf{1}(B \leq B_{1,j}), \\ \hat{g}_1^*(B) &= \frac{1}{m h_g} \sum_{j=1}^m K_g \left( \frac{B - B_{1,j}}{h_g} \right). \end{aligned}$$

## 2.B Linearization of $\Lambda(\cdot)$

Let  $s_k \in \{H, L\}$  be the category of project  $j$ . I then consider the substitution effect of entry by large firms between low- and high-end project. Data exhibit that the amount of high-end contracts is ¥ 851.80 billion, and that of low-end contracts is ¥ 1319.16 billion during the observation period, each of which is computed by the sum of engineer's estimated costs for each project. Due to the fact that the production capacity is likely to be fulfilled only for large firms (see in section 2.3), I assume that the capacity constraint is binding in large firms. Because of the difference in value for each category of projects, the withdrawal of a large firm from high-end projects to bid for low-end projects does not necessarily imply the increase by one more large bidder in low-end projects. This is captured by equation (2.10).

Originally (with the set-aside program), the equilibrium numbers of participants are  $n_{H, BB} = n_{H, BB}^r$  and  $n_{L, BB} = 0$ , so that (2.10) is given by

$$n_{H, BB}^r = \Lambda(0), \quad (\text{A-2})$$

Equation (2.10) also implies that, were the set-aside program to be removed, then for some  $n_{H, BB}^u$  and  $n_{L, BB}^u$

$$n_{H, BB}^u = \Lambda(n_{L, BB}^u). \quad (\text{A-3})$$

The linear approximation of (A-3) evaluated at  $n_{H, BB} = n_{H, BB}^r$  and  $n_{L, BB} = 0$  is thus given by

$$n_{H, BB}^u = \Lambda(0) + \Lambda'(0)\Delta\bar{n}_{L, BB}^u. \quad (\text{A-4})$$

By (A-2) and  $\Delta \bar{n}_{L, BB}^u = \bar{n}_{L, BB}^u$ , one obtains

$$\bar{n}_{H, BB}^u = \bar{n}_{H, BB}^r - \lambda \bar{n}_{L, BB}^u, \quad (\text{A-5})$$

where  $\lambda = -\Lambda'(0)$ . In the counterfactual simulation, I assume that  $\lambda = \frac{851.80}{1319.16}$  (= 0.65).

## 2.C Linearization of $\Gamma(\cdot)$

The linear approximation for  $\bar{n}_{L, SB}^u = \Gamma(\bar{n}_{L, BB}^u)$  at  $\bar{n}_{L, BB}^u = 0$  is given by,

$$\bar{n}_{L, SB}^u = \Gamma(0) - \Gamma'(0) \Delta \bar{n}_{L, BB}^u.$$

Since  $\Gamma(0) = \bar{n}_{L, SB}^r$  and  $\Delta \bar{n}_{L, BB}^u = \bar{n}_{L, BB}^u$ , one obtains

$$\bar{n}_{L, SB}^u = \bar{n}_{L, SB}^r - \gamma \bar{n}_{L, BB}^u. \quad (\text{A-6})$$

To get the explicit form of  $\Gamma(0)$  take total derivative of (2.14) with respect to  $\bar{n}_{L, SB}$  and  $\bar{n}_{L, BB}$ , and one obtains

$$0 = \hat{\alpha}_2 \cdot \frac{\partial \bar{x}_{L, SB}(\cdot)}{\partial \bar{n}_{L, SB}} \Delta \bar{n}_{L, SB} + \hat{\alpha}_2 \cdot \frac{\partial \bar{x}_{L, SB}(\cdot)}{\partial \bar{n}_{L, BB}} \Delta \bar{n}_{L, BB} + \hat{\alpha}_3 \cdot (\Delta \bar{n}_{L, SB} + \Delta \bar{n}_{L, BB}), \quad (\text{A-7})$$

where  $\bar{n}_L = \bar{n}_{L, SB} + \bar{n}_{L, BB}$ .

By the chain rule,  $\frac{\partial \bar{x}_{L, SB}(\cdot)}{\partial \bar{n}_{t, L}} = \frac{\partial \bar{x}_{L, SB}(\cdot)}{\partial \bar{X}_L} \cdot \frac{\partial \bar{X}_L}{\partial \bar{n}_{t, L}}$  holds for each  $t \in \{SB, BB\}$ . Since  $\frac{\partial \bar{x}_{L, SB}(\cdot)}{\partial \bar{X}_L} = -\frac{\bar{X}_{SB}}{(\bar{X}_L)^2}$ ,  $\frac{\partial \bar{X}_L}{\partial \bar{n}_{L, SB}} = 0$  and  $\frac{\partial \bar{X}_L}{\partial \bar{n}_{L, BB}} = \frac{(\bar{X}_{BB} - \bar{X}_{SB}) \bar{n}_{L, SB}^r}{(\bar{n}_L)^2}$  with  $\bar{X}_L = \bar{X}_{SB}$  and

$\bar{n}_L = \bar{n}_{L,SB}^r$ , one obtains

$$\frac{\partial \bar{x}_{L,SB}(\cdot)}{\partial \bar{n}_{L,BB}} = -\frac{\bar{X}_{BB} - \bar{X}_{SB}}{\bar{X}_{SB} \cdot \bar{n}_{L,SB}^r}$$
$$\frac{\partial \bar{x}_{L,SB}(\cdot)}{\partial \bar{n}_{L,SB}} = 0.$$

Plug them into (A-7), and one obtains

$$-\Delta \bar{n}_{L,SB} = \gamma \Delta \bar{n}_{L,BB}, \quad (\text{A-8})$$

where  $\gamma = \frac{\hat{\alpha}_2}{\hat{\alpha}_3} \frac{\bar{X}_{BB} - \bar{X}_{SB}}{\bar{X}_{SB} \cdot \bar{n}_{L,SB}^r} + 1 = 1.32$ .

## APPENDIX FOR CHAPTER 3

### 3.A An alternative way to obtain $1 - F_c(\cdot)$

Since the range of  $\sigma$  is  $[\sigma(\underline{t}), \sigma(\bar{t})]$ , there exists  $\sigma^{-1}(s + \theta_i - \theta)$  if and only if  $\sigma(\underline{t}) \leq s + \theta_i - \theta \leq \sigma(\bar{t})$  for some  $s$ . This implies that his prime contractor beats her rival in the procurement auction with probability  $[1 - F_t(\sigma^{-1}(s + \theta_i - \theta))]^n$  if  $\theta \in [s + \theta_i - \sigma(\bar{t}), s + \theta_i - \sigma(\underline{t})]$  for some  $s$ . On the other hand, if  $s + \theta_i - \sigma(\underline{t}) \leq \theta$  for some  $s$  and  $t_i$ , his prime contractor wins for sure; This happens with probability equal to  $\int_{s + \theta_i - \sigma(\underline{t})}^{\bar{\theta}} f_\theta(\theta) d\theta = 1 - F_\theta[s + \theta_i - \sigma(\underline{t})]$ . Finally, if  $s + \theta_i - \sigma(\bar{t}) \geq \theta$ , his prime contractor will lose.

Let us define  $\sigma(t_{1:n}) = s + \theta_i - \theta$ . Then we have  $\int_{s + \theta_i - \sigma(\bar{t})}^{s + \theta_i - \sigma(\underline{t})} [1 - F_t(\sigma^{-1}(s + \theta_i - \theta))]^n f_\theta(\theta) d\theta = \int_{\sigma(\underline{t})}^{\sigma(\bar{t})} [1 - F_t(\sigma^{-1}(\sigma(t_{1:n})))]^n f_\theta(s + \theta_i - \sigma(t_{1:n})) d\sigma(t_{1:n}) = \int_{\underline{t}}^{\bar{t}} [1 - F_t(\sigma^{-1}(\sigma(t_{1:n})))]^n f_\theta(s + \theta_i - \sigma(t_{1:n})) \sigma'(t_{1:n}) dt_{1:n}$ . Therefore, one obtains

$$1 - F_c(s + \theta_i) = 1 - F_\theta[s + \theta_i - \sigma(\underline{t})] + \int_{\underline{t}}^{\bar{t}} [1 - F_t(t_{1:n})]^n f_\theta(s + \theta_i - \sigma(t_{1:n})) dt_{1:n}.$$

From integral by parts

$$\begin{aligned}
1 - F_c(s + \theta_i) &= 1 - F_\theta[s + \theta_i - \sigma(\underline{t})] - \left[ [1 - F_t(t_{1:n})]^n F_\theta(s + \theta_i - \sigma(t_{1:n})) \right]_{\underline{t}}^{\bar{t}} \\
&\quad - \int_{\underline{t}}^{\bar{t}} n f_t [1 - F_t(t_{1:n})]^{n-1} F_\theta(s + \theta_i - \sigma(t_{1:n})) dt_{1:n} \\
&= \int_{\underline{t}}^{\bar{t}} n f_t [1 - F_t(t_{1:n})]^{n-1} [1 - F_\theta(s + \theta_i - \sigma(t_{1:n}))] dt_{1:n},
\end{aligned}$$

where the last equality holds from the fact that  $\int_{\underline{t}}^{\bar{t}} n f_t [1 - F_t(t_{1:n})]^{n-1} dt_{1:n} = 1$ .

### 3.B Proof that $F_c$ has IHR

*Proof.* Define  $\delta \equiv c_{1:N-1} - \theta_i$ , where  $c_{1:N-1}$  denotes the lowest order statistics among  $N - 1$  iid valuation samples of  $c$ . Since, by (A-2),  $-\frac{Q'(s|\cdot)}{Q(s|\cdot)} \equiv \frac{f_\delta(s|\cdot)}{1 - F_\delta(s|\cdot)}$  is increasing in  $s$ ,  $F_\delta$  has IHR. Now we consider the random variable  $c_{1:N-1} = \delta + \theta_i$ . Since the IHR is closed in convolution,  $F_{c_{1:N-1}}$  has IHR. Then, from (A-2),  $F_c$  has IHR if and only if  $F_{c_{1:N-1}}$  has IHR. Hence,  $\frac{f_c}{1 - F_c}$  is increasing.  $\square$

### 3.C Proof that $Q$ has IHR

*Proof.* Show that  $Q(s)$  has monotone increasing hazard rate (IHR). Equivalently, I show that  $-\frac{Q'(s|\cdot)}{Q(s|\cdot)}$  is increasing in  $s$ . Since  $s = \sigma(t)$ , we have  $F_\sigma(s) = F_t(\sigma^{-1}(s))$ . Then, we have

$$\frac{f_\sigma(s)}{1 - F_\sigma(s)} = \frac{f_t(\sigma^{-1}(s))}{1 - F_t(\sigma^{-1}(s))} \frac{1}{\sigma'(\sigma^{-1}(s))}. \quad (\text{A-1})$$



On the other hand, from the bidding function

$$\sigma'(\sigma^{-1}(s)) = (n-1) \frac{f_t(\sigma^{-1}(s))}{1 - F_t(\sigma^{-1}(s))} \frac{1}{\frac{1}{s - \sigma^{-1}(s)} + \frac{Q'(s|\cdot)}{Q(s|\cdot)}},$$

or equivalently,

$$\frac{1}{s - \sigma^{-1}(s)} + \frac{Q'(s|\cdot)}{Q(s|\cdot)} = (n-1) \frac{f_t(\sigma^{-1}(s))}{1 - F_t(\sigma^{-1}(s))} \frac{1}{\sigma'(\sigma^{-1}(s))}$$

Then, we suppose by contradiction that  $\frac{Q'(s|\cdot)}{Q(s|\cdot)}$  is increasing in  $s$ . Given that  $s - \sigma^{-1}(s)$  is decreasing in  $s$ , the right hand side must be increasing in  $s$ . Substituting it into the right hand side of (A-1) implies that  $F_\sigma$  must have monotone increasing hazard rate (IHR). On the other hand, log-concavity of  $f_\theta$  implies IHR of  $F_\theta$ . Since the convolution of two random variables with IHR is also IHR (Barlow and Proschan (1975)[1, Sect 4.4]), the distribution of the random variable  $c \equiv \theta + \sigma(t)$  must be IHR. Now, let  $c_{1:N-1}$  denote the lowest order statistics among  $N - 1$  iid valuation samples of  $c$ . Then, the hazard rate of the random variable  $c_{1:N-1}$  is given by

$$\frac{(N-1)f_c(c_{1:N-1})[1 - F_c(c_{1:N-1})]^{N-2}}{[1 - F_c(c_{1:N-1})]^{N-1}} = \frac{(N-1)f_c(c_{1:N-1})}{[1 - F_c(c_{1:N-1})]},$$

which is increasing in  $c_{1:N-1}$  if and only if  $F_c$  is IHR. Thus, the cumulative distribution of  $c_{1:N-1}$ , *i.e.*  $F_{c_{1:N-1}}$ , must also be IHR. Finally, define  $\delta \equiv c_{1:N-1} - \theta_i$ . Since  $F_{c_{1:N-1}}$  and  $F_\theta$  are IHR, the convolution of these two random variables  $\delta$  is also IHR. Note that  $Q(s|\cdot)$  is the probability that  $\Pr\{s \leq \delta\}$ . Hence, denoting by  $F_\delta$  the cumulative distribution function of  $\delta$ , we can rewrite  $Q(s|\cdot)$  as  $1 - F_\delta(s|\cdot)$  and

$Q'(s|\cdot)$  as  $-f_\delta(s|\cdot)$ . Since  $F_\delta$  has IHR, we must have

$$\frac{-f_\delta(s|\cdot)}{1 - F_\delta(s|\cdot)} = \frac{Q'(s|\cdot)}{Q(s|\cdot)}$$

is decreasing. We reached a contradiction. □

## APPENDIX FOR CHAPTER 4

### 4.A Proof for Lemma 4

Since  $\partial^2 S(p|x^*, \cdot)/\partial p^2 < 0$ , the first order principle minor of the Hessian on  $S(p|\cdot)$  is negative.

Take second order derivative on  $S$ .

$$\begin{aligned} \frac{\partial^2 S(p|\cdot)}{\partial p_1^2} &= N_1(N_1-1) \cdot \sum_{\hat{n}_1=0}^{N_1-2} \sum_{\hat{n}_2=0}^{N_2} P_{\hat{n},p}^{N_1-2, N_2} [V(n_1+2, n_2|\cdot) - 2V(n_1+1, n_2|\cdot) + V(n_1, n_2|\cdot)] \\ \frac{\partial^2 S(p|\cdot)}{\partial p_1 \partial p_2} &= N_1 N_2 \cdot \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n},p}^{N_1-1, N_2-1} [V(n_1+1, n_2+1|\cdot) - V(n_1, n_2+1|\cdot) \\ &\quad - V(n_1+1, n_2|\cdot) + V(n_1, n_2|\cdot)] \\ \frac{\partial^2 S(p|\cdot)}{\partial p_2^2} &= N_2(N_2-1) \cdot \sum_{\hat{n}_1=0}^{N_1} \sum_{\hat{n}_2=0}^{N_2-2} P_{\hat{n},p}^{N_1, N_2-2} [V(n_1, n_2+2|\cdot) - 2V(n_1, n_2+1|\cdot) + V(n_1, n_2|\cdot)] \end{aligned}$$

The second order principal minor of the Hessian on  $S(p_1, p_2|\cdot)$ , which is given as  $(N_1 - 1)(N_2 - 1) \cdot \sum_{\hat{n}_1=0}^{N_1-2} \sum_{\hat{n}_2=0}^{N_2} P_{\hat{n},p}^{N_1-2, N_2} [\pi_1(n_1, n_2 + 2|x) - \pi_1(n_1, n_2 + 1|x)] \cdot \sum_{\hat{n}_1=0}^{N_1} \sum_{\hat{n}_2=0}^{N_2-2} P_{\hat{n},p}^{N_1, N_2-2} [\pi_2(n_1+2, n_2|x) - \pi_2(n_1+1, n_2|x)] - N_1 N_2 \cdot \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n},p}^{N_1-1, N_2-1} [\pi_1(n_1, n_2 + 1|x) - \pi_1(n_1, n_2|x)] \cdot \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n},p}^{N_1-1, N_2-1} [\pi_2(n_1+1, n_2|x) - \pi_2(n_1, n_2|x)]$  by Lemma 2, is positive if and only if  $G_1(p|\cdot) - G_2(p|\cdot) > 0$ . Hence, the even equilibrium is a saddle point. Finally, I show  $S(p_1, p_2|x^*, U_1(p_1, p_2) = 0)$  is increasing in  $p_1$  if and only if  $U_1(p_1, p_2|x^*) > U_2(p_1, p_2|x^*)$ . Suppose, by contradiction, there exist  $p'_1$  and  $p'_2$

such that  $S(p'_1, p'_2|x^*)|_{A_1=0}$  is decreasing in  $p_1$ . Since  $U_2(\cdot)$  is continuous in  $p_2$ , there exists some  $p''_2 < p'_2$  such that  $U_2(p'_1, p''_2) = 0$ . By (4.8),  $S(p'_1, p''_2|x^*) > S(p'_1, p_2|x^*)$  for any  $p_2$ . Therefore,  $S(p'_1, p''_2|x^*) > S(p'_1, p'_2|x^*)$ . Since  $U_1(\cdot)$  is continuous in  $p_1$ , there exists  $p'' > p'$  such that  $U_1(p'', p''_2) = 0$ . By (4.8),  $S(p'', p''_2|x^*) > S(p_1, p''_2|x^*)$  for any  $p_1$ . Therefore,  $S(p'', p''_2|x^*) > S(p'_1, p''_2|x^*)$ . Hence,  $S(p'', p''_2|x^*) > S(p'_1, p'_2|x^*)$ , which leads to contradiction.

## 4.B Proof for Lemma 5

Suppose contradictory that the set  $\hat{Y}$  is empty. Set  $y_1 = \tilde{y}_1$  such that  $u_1(n_1 + 1, n_2 + 1|x_1, \tilde{y}_1) - u_1(n_1 + 1, n_2|x_1, \tilde{y}_1) = 0$  and  $u_1(n_1 + 2, n_2|x_1, \tilde{y}_1) - u_1(n_1 + 1, n_2|x_1, \tilde{y}_1) > 0$ . Then,  $G_1(p|x_1, \tilde{y}_1) = 0$  for any  $p$ . Also, set  $y_2 = \tilde{y}_2$  such that  $u_2(n_1, n_2 + 2|x_2, \tilde{y}_2) - u_2(n_1, n_2 + 1|x_2, \tilde{y}_2) > 0$  and  $u_2(n_1 + 1, n_2 + 1|x_2, \tilde{y}_2) - u_2(n_1 + 1, n_2|x_2, \tilde{y}_2) = 0$ . Then,  $G_2(p|x_2, \tilde{y}_2) \rightarrow \infty$  for any  $p$ . Since  $G_1 < G_2$ ,  $\tilde{y} = (\tilde{y}_1, \tilde{y}_2) \in \hat{Y}$ , which contradicts. Thus  $\hat{Y}$  is nonempty.

Let  $\hat{y} \in \hat{Y}$ . Set  $y^0_\tau(p) = -U_\tau(p|x_\tau, \hat{y}_\tau)$  for some  $p$ . Then,  $U_\tau(p|x_\tau, \hat{y}_\tau + y^0_\tau(p)) = 0$  for any  $p$ . Since  $y^0_\tau(p)$  is constant for all  $n$ ,  $G_1(p|x_1, \hat{y}_1 + y^0_1) < G_2(p|x_2, \hat{y}_2 + y^0_2)$  for any  $x$ . Hence,  $\hat{y}(p) + y^0(p) \in \hat{Y}$ .

## 4.C Proof for Theorem 2

*Proof.* Suppose there exists  $b_\tau(v|x)$  for some  $x$  such that the bidder who has the highest  $b$  wins the item. Define  $\phi_\tau(v|x) = b^{-1}_\tau(b_\tau(v|x)|x)$  and  $\hat{F}_\tau(v|x) = F_\tau(\phi_{-\tau}(v|x))$ .

Then, (4.2) becomes

$$\begin{aligned}
\pi_1(n_1, n_2|x) &= \int_v (1-F_1(v))(F_1(v))^{n_1-1}(\hat{F}_2(v|x))^{n_2} dv, \\
&= \int_v (F_1(v))^{n_1-1}(\hat{F}_2(v|x))^{n_2} dv - \int_v (F_1(v))^{n_1}(\hat{F}_2(v|x))^{n_2} dv, \\
&= \int_v v[(F_1(v))^{n_1}(\hat{F}_2(v|x))^{n_2}]' dv - \int_v v[(F_1(v))^{n_1-1}(\hat{F}_2(v|x))^{n_2}]' dv.
\end{aligned}$$

where  $F_\tau(\cdot) = v_\tau^{-1}(\cdot)$  and  $f_\tau(\cdot) = F'_\tau(\cdot)$ . The last equality holds from integral by parts. Since  $\hat{F}_1(v|x) = F_1(\phi_2(v))$ ,  $\hat{F}_1(\phi_1(v)) = F_1(\phi_2(\phi_1(v))) = F_1(v)$  and  $\phi_2(\phi_1(v)) = v_1(v)$ , letting  $\phi_1(v) = \hat{v}$ , one obtains

$$\begin{aligned}
\int v(F_1(v))^{n_1}[(\hat{F}_2(v|x))^{n_2}]' dv &= \int \phi_2(\phi_1(v)) \cdot [\hat{F}_1(\phi_1(v))]^{n_1} n_2 f_2(\phi_1(v))(F_2(\phi_1(v)))^{n_2-1} \phi_1'(v) dv \\
&= \int_{\hat{v}} \phi_2(\hat{v}) \cdot [\hat{F}_1(\hat{v})]^{n_1} n_2 f_2(\phi_1(v))(F_2(\phi_1(v)))^{n_2-1} d\hat{v}.
\end{aligned}$$

Recall

$$\begin{aligned}
V(n_1-1, n_2|x) &= \int v(n_1-1)f_1(v)(F_1(v))^{n_1-2}(\hat{F}_2(v|x))^{n_2} dv + \int v(\hat{F}_1(v|x))^{n_1-1} n_2 f_2(v)(F_2(v))^{n_2-1} dv \\
V(n_1, n_2|x) &= \int v n_1 f_1(v)(F_1(v))^{n_1-1}(\hat{F}_2(v|x))^{n_2} dv + \int v(\hat{F}_1(v|x))^{n_1} n_2 f_2(v)(F_2(v))^{n_2-1} dv.
\end{aligned}$$

Therefore,

$$V(n_1, n_2|x) - V(n_1-1, n_2|x) + \int_v [\phi_2(v) - v][1 - \hat{F}_1(v|x)](\hat{F}_1(v|x))^{n_1-1}[(F_2(v))^{n_2}]' dv = \pi_1(n_1, n_2|x).$$

$\phi_\tau(v) = v$  for any  $v$  under the *ex post* efficient mechanism. Therefore, the third term on the left hand side vanishes if  $x = x^*$ . Clearly, the term is typically non zero for any  $x \in X \setminus \{x^*\}$ .  $\square$

#### 4.D Proof for Proposition 9

Define  $H_\tau(p_{-\tau}|\cdot) : [0, 1] \rightarrow [0, 1]$  as the solution of (4.4) for  $p_\tau$ . Since  $U_\tau^i$  is decreasing in  $p$ , we have  $p'_{-\tau} \leq p''_{-\tau} \leq p'''_{-\tau}$ , where  $p'_{-\tau} \in \{p_{-\tau}|U_\tau^i(1, p_{-\tau}|\cdot) > 0\}$ ,  $p''_{-\tau} \in [0, 1] \setminus \{p_{-\tau}|U_\tau^i(1, p_{-\tau}|\cdot) > 0\} \cup \{p_{-\tau}|U_\tau^i(0, p_{-\tau}|\cdot) < 0\}$ , and  $p'''_{-\tau} \in \{p_{-\tau}|U_\tau^i(0, p_{-\tau}|\cdot) < 0\}$ . Therefore,  $H_\tau$  can be described as

$$H_\tau(p_{-\tau}|\cdot) = \begin{cases} 1 & \text{if } \{p_{-\tau}|U_\tau^i(1, p_{-\tau}|\cdot) > 0\} \\ 0 & \text{if } \{p_{-\tau}|U_\tau^i(0, p_{-\tau}|\cdot) < 0\} \\ \{p_\tau|U_\tau^i(p_1, p_2|\cdot) = 0\} & \text{otherwise.} \end{cases} \quad (\text{A-1})$$

Since  $(p_1^*, p_2^*)$  must satisfy (A-1), we have

$$\begin{cases} H_1(p_2^*|x_1, y_1) = p_1^*, \\ H_2(p_1^*|x_2, y_2) = p_2^*. \end{cases}$$

That is, if we define  $H(p|\cdot) = \times_{\tau \in \{1, 2\}} H_\tau(p_{-\tau}|\cdot)$ , a type symmetric equilibrium  $p^* = (p_1^*, p_2^*)$  is a fixed point of  $H$  i.e.,  $p^* \in H(p^*|x, y)$ .

$u_\tau(n|\cdot)$  is decreasing in  $n$ . Since  $n$  follows a binomial distribution,  $U_\tau$  is continuous. Hence, by (A-1),  $H_\tau(p_{-\tau}|\cdot)$  is continuous. For each  $\tau$  the set  $H_\tau(p_{-\tau})$  is nonempty and has a closed graph since  $H_\tau(\cdot)$  is continuous. Thus, by the fixed point theorem, there exists at least one fixed point on  $H$ . A mixed-strategy equilibrium is a fixed point.

## 4.E Proof for Proposition 10

By the characteristic of binomial distribution, the following identities hold:

$$\left\{ \begin{array}{l} \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2} P_{\hat{n},p}^{N_1-1,N_2} u_1(\hat{n}_1+1, \hat{n}_2|\cdot) \\ \equiv \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n},p}^{N_1-1,N_2-1} [p_2 \cdot u_1(\hat{n}_1+1, \hat{n}_2+1|\cdot) + (1-p_2) \cdot u_1(\hat{n}_1+1, \hat{n}_2|\cdot)] \\ \sum_{\hat{n}_1=0}^{N_1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n},p}^{N_1,N_2-1} u_2(\hat{n}_1, \hat{n}_2+1|\cdot) \\ \equiv \sum_{\hat{n}_1=0}^{N_1-1} \sum_{\hat{n}_2=0}^{N_2-1} P_{\hat{n},p}^{N_1-1,N_2-1} [p_1 \cdot u_2(\hat{n}_1+1, \hat{n}_2+1|\cdot) + (1-p_1) \cdot u_2(\hat{n}_1, \hat{n}_2+1|\cdot)]. \end{array} \right. \quad (\text{A-2})$$

If  $U_1 = U_2$ , then by monotonicity,

$$\begin{aligned} p_2 \sum_{\hat{n}_1} \sum_{\hat{n}_2} P_{\hat{n}} u_1(\hat{n}_1+1, \hat{n}_2+1) + (1-p_2) \sum_{\hat{n}_1} \sum_{\hat{n}_2} P_{\hat{n}} u_1(\hat{n}_1+1, \hat{n}_2) \\ \leq p_1 \sum_{\hat{n}_1} \sum_{\hat{n}_2} P_{\hat{n}} u_1(\hat{n}_1+1, \hat{n}_2+1) + (1-p_1) \sum_{\hat{n}_1} \sum_{\hat{n}_2} P_{\hat{n}} u_1(\hat{n}_1+1, \hat{n}_2). \end{aligned}$$

Hence,  $(p_2 - p_1) \sum_{\hat{n}_1} \sum_{\hat{n}_2} P_{\hat{n}} u_1(\hat{n}_1+1, \hat{n}_2+1) \leq (p_2 - p_1) \sum_{\hat{n}_1} \sum_{\hat{n}_2} P_{\hat{n}} u_1(\hat{n}_1+1, \hat{n}_2)$ .  
 Since, for any  $p_1$  and  $p_2$ ,  $\sum_{\hat{n}_1} \sum_{\hat{n}_2} P_{\hat{n}} u_1(\hat{n}_1+1, \hat{n}_2+1) - \sum_{\hat{n}_1} \sum_{\hat{n}_2} P_{\hat{n}} u_1(\hat{n}_1+1, \hat{n}_2) < 0$ ,  
 one must obtain  $p_2 \geq p_1$ . Equality holds if both  $u_1(n_1+1, n_2+1) = u_2(n_1+1, n_2+1)$   
 and  $u_1(n_1+1, n_2) = u_2(n_1, n_2+1)$  hold for all  $n$ .

## BIBLIOGRAPHY

- Athey, Susan and Philip A. Haile**, “Empirical Models of Auctions,” Levine’s Bibliography 12224700000001045, UCLA Department of Economics March 2006.
- Ayres, Ian and Peter Cramton**, “Deficit Reduction Through Diversity: How Affirmative Action at the FCC Increased Auction Competition,” Technical Report 1996.
- Bajari, Patrick and Steven Tadelis**, “Incentives versus Transaction Costs: A Theory of Procurement Contracts,” *RAND Journal of Economics*, Autumn 2001, 32 (3), 387–407.
- Bannock, Graham**, *The Economics of Small Firms: Return from the Wilderness*, Oxford: Basil Blackwell, 1981.
- Barlow, R. and F. Proschan**, *Statistical Theory of Reliability and Life Testing Probability Models*, Holt, Rinehart and Winston Inc., 1975.
- Bulow, Jeremy and John Roberts**, “The Simple Economics of Optimal Auctions,” *Journal of Political Economy*, October 1989, 97 (5), 1060–90.
- Campo, Sandra, Isabelle Perrigne, and Quang Vuong**, “Asymmetry in first-price auctions with affiliated private values,” *Journal of Applied Econometrics*, 2003, 18 (2), 179–207.



- Corns, Allan and Andrew Schotter**, “Can Affirmative Action Be Cost Effective? An Experimental Examination of Price-Preference Auctions,” *American Economic Review*, March 1999, 89 (1), 291–305.
- Degn, Eric and Kevin R. Miller**, “Bid Shopping,” *Journal of Construction Education*, 2003, 8 (1), 47–55.
- Denes, Thomas A.**, “Do Small Business Set-Asides Increase the Cost of Government Contracting?,” *Public Administration Review*, 1997, 57 (5), 441–444.
- Dyer, Douglas and John H. Kagel**, “Bidding in Common Value Auctions: How the Commercial Construction Industry Corrects for the Winner’s Curse,” *Management Science*, October 1996, 42 (10), 1463–1475.
- Engelbrecht-Wiggans, Richard**, “Optimal Auctions Revisited,” *Games and Economic Behavior*, April 1993, 5 (2), 227–239.
- Gale, Ian L, Donald B Hausch, and Mark Stegeman**, “Sequential Procurement with Subcontracting,” *International Economic Review*, November 2000, 41 (4), 989–1020.
- Gilbert, Richard J. and Paul Klemperer**, “An Equilibrium Theory of Rationing,” *RAND Journal of Economics*, Spring 2000, 31 (1), 1–21.
- Grosskopf, Ofer and Barak Medina**, “Rationalizing Drennan: On Irrevocable Offers, Bid Shopping and Binding Range,” *Review of Law & Economics*, 2007, 3 (2), 4.

- Guerre, Emmanuel, Isabelle Perrigne, and Quang Vuong**, “Optimal Non-parametric Estimation of First-Price Auctions,” *Econometrica*, 2000, 68 (3), 525–574.
- Hansen, Robert G.**, “Auctions with Endogenous Quantity,” *RAND Journal of Economics*, Spring 1988, 19 (1), 44–58.
- Kamien, Morton I., Lode Li, and Dov Samet**, “Bertrand Competition with Subcontracting,” *RAND Journal of Economics*, Winter 1989, 20 (4), 553–567.
- Kawasaki, Seiichi and John McMillan**, “The design of contracts: Evidence from Japanese subcontracting,” *Journal of the Japanese and International Economies*, September 1987, 1 (3), 327–349.
- Kjerstad, Egil and Steinar Vagstad**, “Procurement auctions with entry of bidders,” *International Journal of Industrial Organization*, December 2000, 18 (8), 1243–1257.
- Laffont, Jean-Jacques and Jean Tirole**, *A theory of incentives in procurement and regulation*, Cambridge, MA: The MIT Press, 1994.
- Lebrun, Bernard**, “First Price Auctions in the Asymmetric N Bidder Case,” *International Economic Review*, February 1999, 40 (1), 125–42.
- Levin, Dan and James L Smith**, “Equilibrium in Auctions with Entry,” *American Economic Review*, June 1994, 84 (3), 585–99.
- Li, Tong and Xiaoyong Zheng**, “Entry and competition effects in first-price auctions: theory and evidence from procurement auctions,” CeMMAP working

papers CWP13/06, Centre for Microdata Methods and Practice, Institute for Fiscal Studies August 2006.

**Marechal, Francois and Pierre-Henri Morand**, “Pre vs. post-award subcontracting plans in procurement bidding,” *Economics Letters*, October 2003, *81* (1), 23–30.

**Marion, Justin**, “Are Bid Preferences Benign? The Effect of Small Business Subsidies in Highway Procurement Auctions,” December 2004. working paper, University of California at Santa Cruz.

**Maskin, Eric and John Riley**, “Asymmetric Auctions,” *Review of Economic Studies*, July 2000, *67* (3), 413–38.

**McAfee, R. Preston and John McMillan**, “Auctions and Bidding,” *Journal of Economic Literature*, June 1987, *25* (2), 699–738.

– **and** – , “Auctions with entry,” *Economics Letters*, 1987, *23* (4), 343–347.

– **and** – , “Government procurement and international trade,” *Journal of International Economics*, May 1989, *26* (3-4), 291–308.

**Milgrom, Paul**, *Putting Auction Theory to Work*, Cambridge University Press, 2004.

**Milgrom, Paul R and Robert J Weber**, “A Theory of Auctions and Competitive Bidding,” *Econometrica*, September 1982, *50* (5), 1089–1122.

- MLIT, Ministry of Land, Infrastructure, Transportation and Tourism,**  
*Construction Contracts: Annual Report: Tyokkatsu Koji Keiyaku Kankei Shiryou*  
 2007. available at <http://www.mlit.go.jp/chotatsu/contractsystem/keiyaku/>.
- Myerson, Roger B,** “Optimal auction design,” *Mathematics of Operations Research*, 1981, 6 (1), 58–73.
- Pevnitskaya, Svetlana,** “Endogenous Entry and Self-selection in Private Value Auctions: An Experimental Study,” 2003. working paper, California Institute of Technology, Division of the Humanities and Social Sciences.
- , “Endogenous Entry in First-Price Private Value Auctions: the Self-Selection Effect,” 2003. working paper, Ohio State University.
- Riley, John G and William F Samuelson,** “Optimal Auctions,” *American Economic Review*, June 1981, 71 (3), 381–92.
- Samuelson, William F.,** “Competitive bidding with entry costs,” *Economics Letters*, 1985, 17 (1-2), 53–57.
- Vickrey, William,** “Counterspeculation, Auctions, and Competitive Sealed Tenders,” *Journal of Finance*, Mar 1961, 16 (1), 8–37.
- Ye, Lixin,** “Optimal Auctions with Endogenous Entry,” *Contributions to Theoretical Economics*, 2004, 4 (1), 1158–1158.
- Zhang, Bin and Kemal Guler,** “Nonparametric estimation of asymmetric first price auctions: A simplified approach,” *Economics Letters*, September 2005, 88 (3), 318–322.

**Zheng, Charles Z.**, “High Bids and Broke Winners,” *Journal of Economic Theory*,  
September 2001, *100* (1), 129–171.